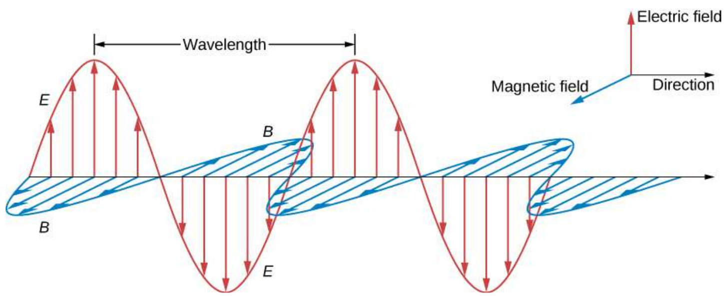
Quantum noise of GW detector

Quantum noise of light

Quantum Shot noise

-> Noise induced by photon

$$Itensity \rightarrow \left|\vec{E}\right|^2 \rightarrow (W/m^2) \propto (Number\ of\ photon)$$

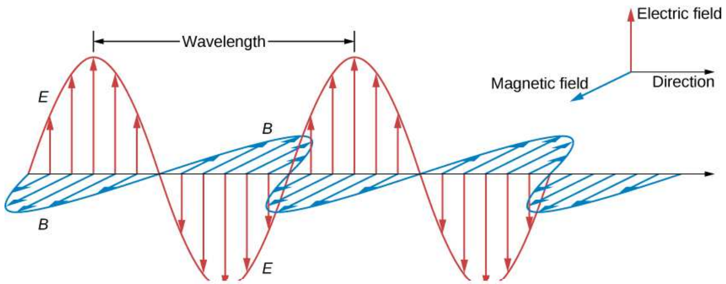


$$Photo\ dioe\ current\ (I) \propto Intensity \propto (Number\ of\ photon)$$

Classical electric field

Quantum Shot noise
-> Noise induced by photon

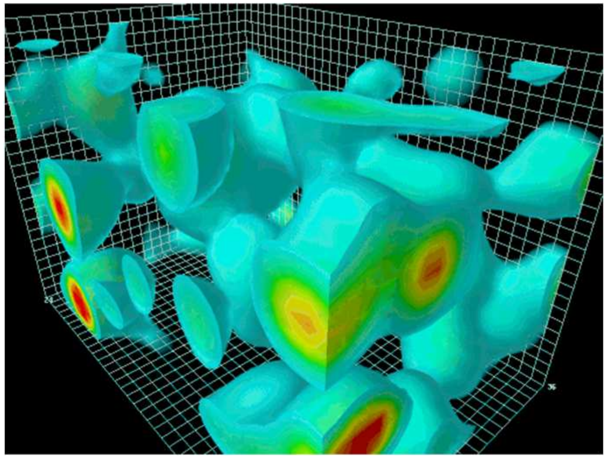
$$Itensity \rightarrow |\vec{E}|^2 \rightarrow (W/m^2) \propto (Number\ of\ photon)$$



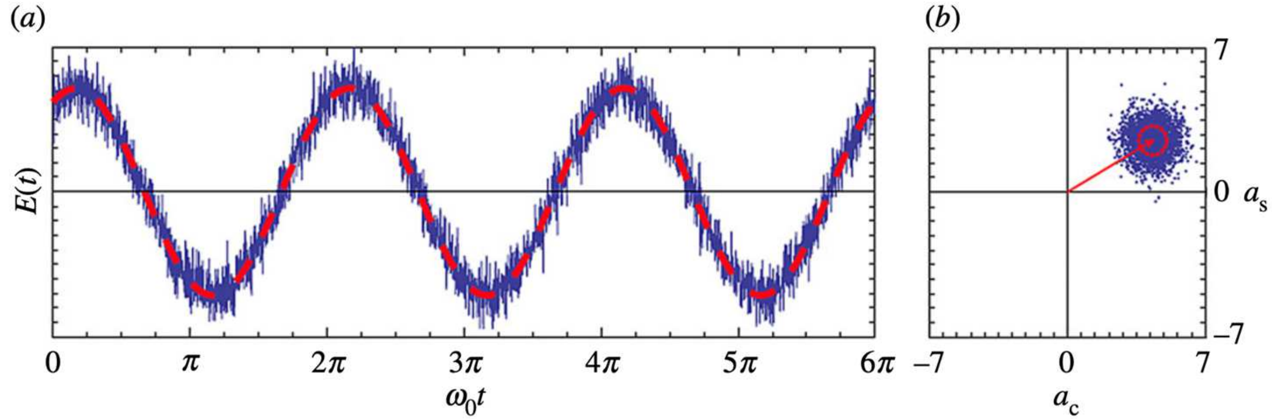
$$E_{1,r}(t) \approx \frac{1}{\sqrt{2}}[E_0\cos(\omega_0 t) - E_0\sin(\omega_0 t)\frac{2\omega_0 x_1(t)}{c} + E_{as}(t)]$$

$$E_{2,r}(t) \approx \frac{1}{\sqrt{2}}[E_0\cos(\omega_0 t) - E_0\sin(\omega_0 t)\frac{2\omega_0 x_2(t)}{c} - E_{as}(t)]$$

Vacuum fluctuation

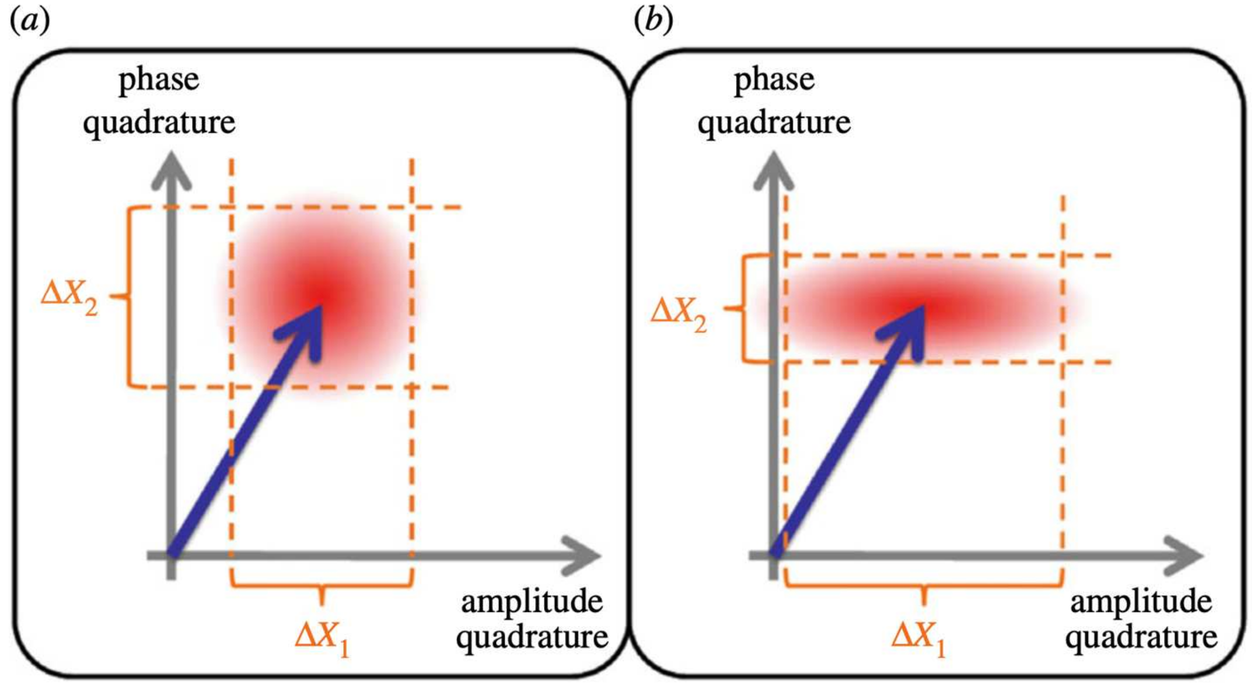


Real electric field of light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit.Phil. Trans. R. Soc. A 376: 20170289.

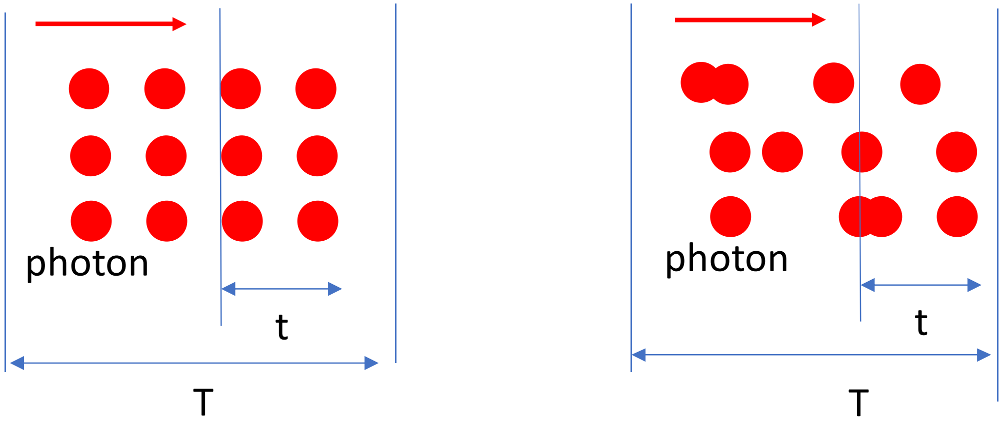
Phase and amplitude noise of light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit.Phil. Trans. R. Soc. A 376: 20170289.

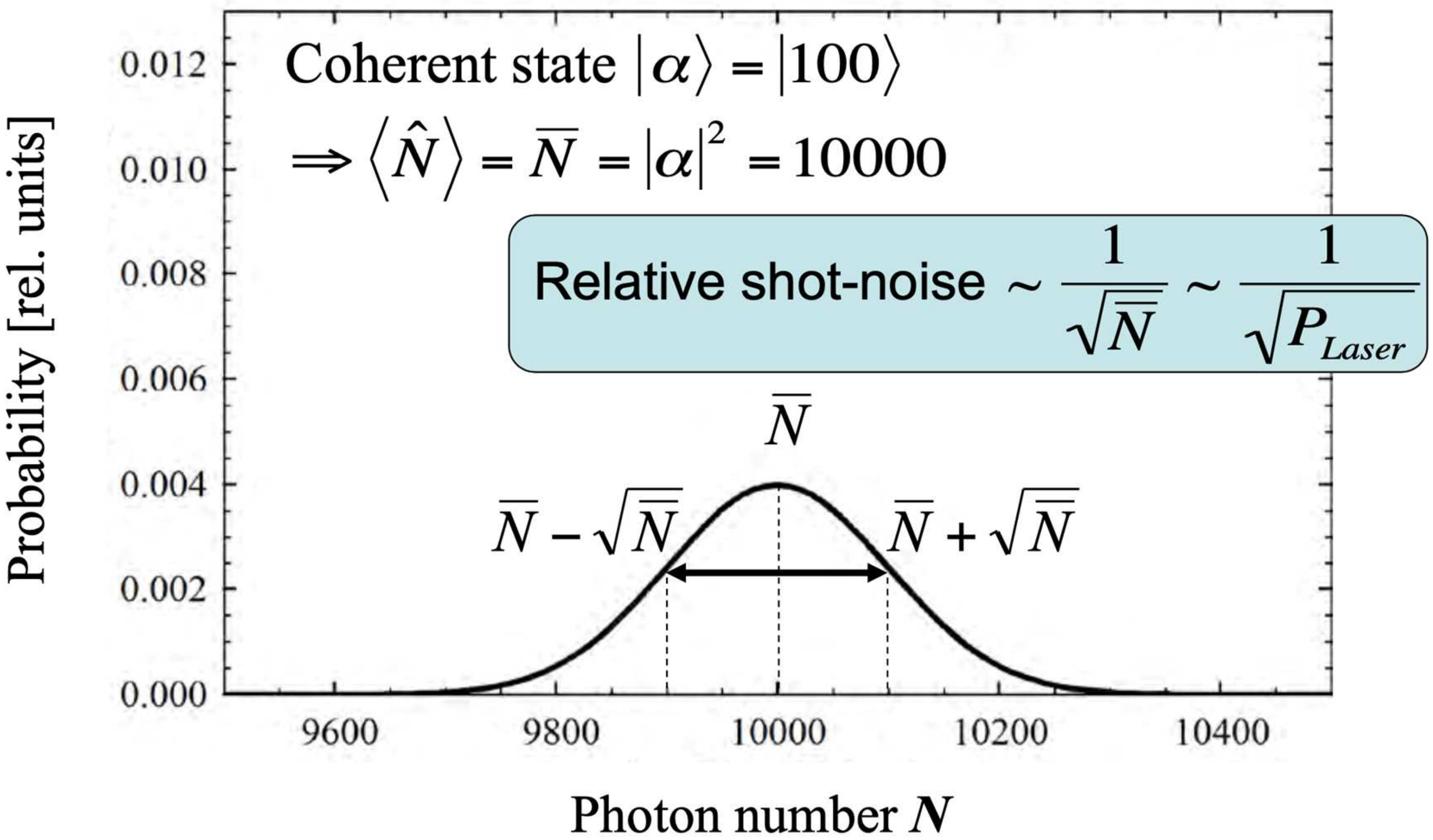
Shot noise of interferometer

Laser power = the number of photon / time

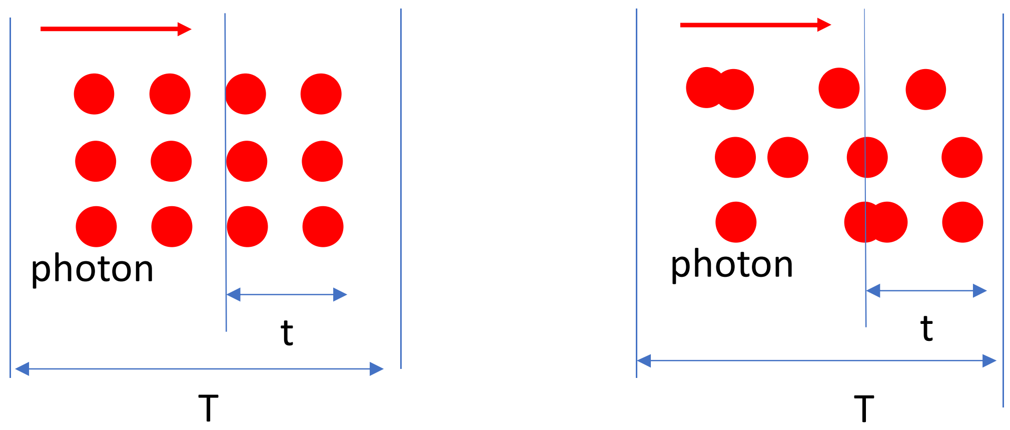


Shot noise

Photon Counting Statistics



Shot noise of interferometer



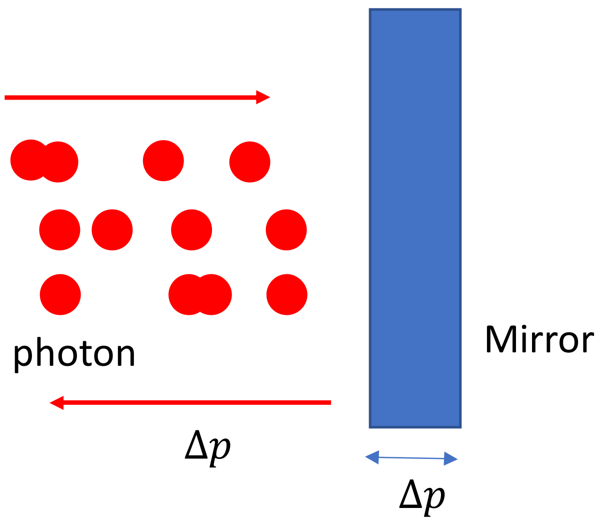
Shot noise

When we have coherent laser source

$$\frac{N}{\Delta N} = {}^{N}/_{\sqrt{N}} \qquad SNR\ by\ photon = N/\sqrt{N}$$

If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
We say that it has shot noise limit sensitivity

Radiation pressure noise



- Stored energy is very high (750 kW)
- Desired sensitivity is very high ($10^{-21} \sim 10^{-24}$)

Electric field in simple Michelson interferometer

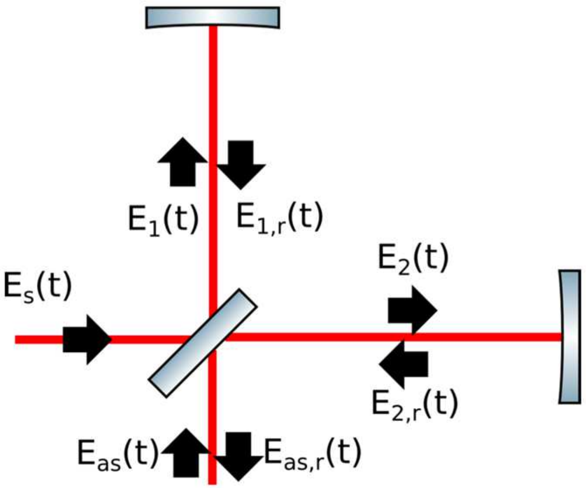


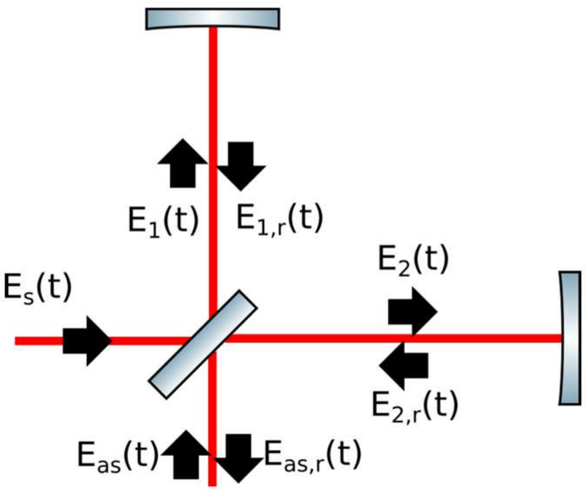
Figure 1-3: Schematic of a Michelson interferometer. A classical carrier field $E_s(t)$ enters from the interferometer symmetric port while a vacuum fluctuations represented by $E_{as}(t)$ enter from the anti-symmetric port. The quantum noise level at the readout is contained in the AC component of the field exiting the interferometer $E_{as,r}(t)$

$$E_s(t) = E_0 \cos(\omega_0 t) + \underline{\delta E_s(t)}$$

Noise term

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,
University of California Berkeley, Eric Oelker (2009)

Electric field in simple Michelson interferometer



Assume anti-symmetric port is dark port

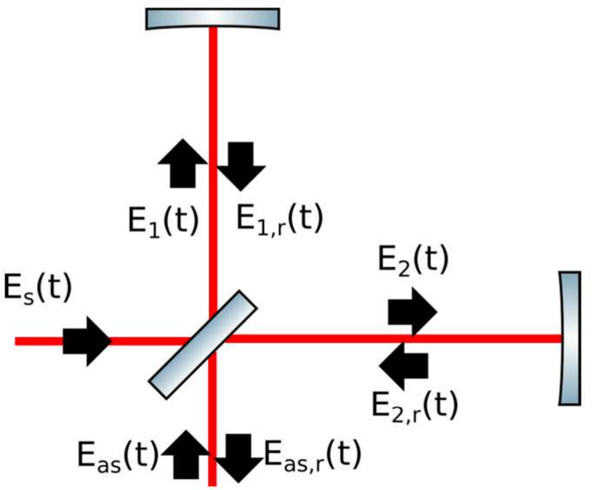
$$E_1(t) = \frac{1}{\sqrt{2}}[E_s(t) + E_{as}(t)]$$

$$E_2(t) = \frac{1}{\sqrt{2}}[E_s(t) - E_{as}(t)]$$

'as' is vacuum field

Squeezed States for Advanced Gravitational Wave Detectors, B.A., University of California Berkeley, Eric Oelker (2009)

Electric field in simple Michelson interferometer

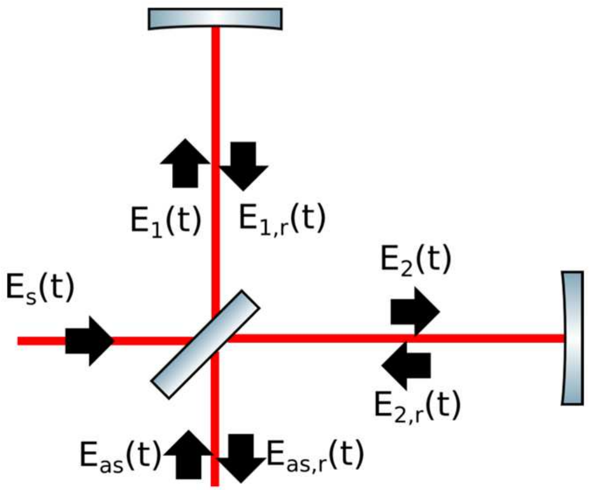


$$E_{1,r}(t) = \frac{1}{\sqrt{2}}[E_s(t - \frac{2x_1(t)}{c}) + E_{as}(t - \frac{2x_1(t)}{c})]$$

$$E_{2,r}(t) = \frac{1}{\sqrt{2}}[E_s(t - \frac{2x_2(t)}{c}) - E_{as}(t - \frac{2x_2(t)}{c})]$$

Squeezed States for Advanced Gravitational Wave Detectors, B.A., University of California Berkeley, Eric Oelker (2009)

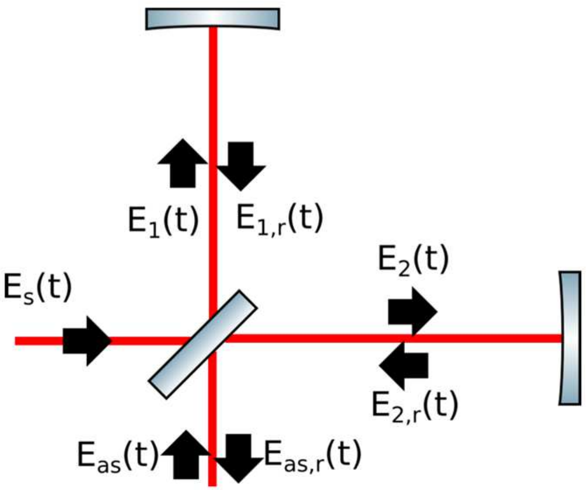
Electric field in simple Michelson interferometer



$$E_{1,r}(t) \approx \frac{1}{\sqrt{2}}[E_0 \cos(\omega_0 t) - E_0 \sin(\omega_0 t)\frac{2\omega_0 x_1(t)}{c} + E_{as}(t)]$$
$$E_{2,r}(t) \approx \frac{1}{\sqrt{2}}[E_0 \cos(\omega_0 t) - E_0 \sin(\omega_0 t)\frac{2\omega_0 x_2(t)}{c} - E_{as}(t)]$$

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,
University of California Berkeley, Eric Oelker (2009)

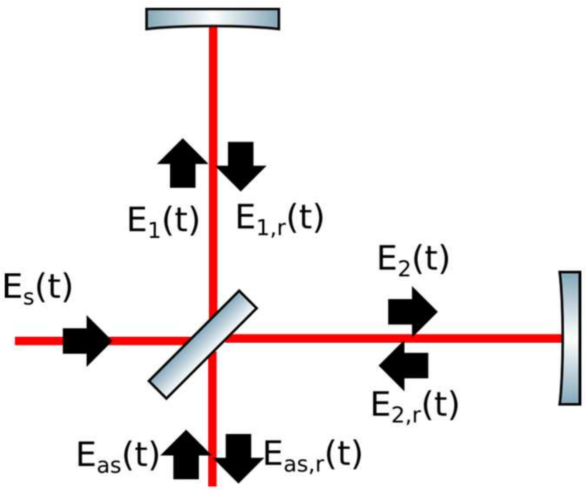
Electric field in simple Michelson interferometer



$$E_{as,r}(t) = E_{as} + E_0 \frac{\omega_0[x_2(t) - x_1(t)]}{c} \sin(\omega_0 t)$$

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,
University of California Berkeley, Eric Oelker (2009)

Electric field in simple Michelson interferometer



$$E_{as,r}(t) = E_{as} + E_0 \frac{\omega_0 [x_2(t) - x_1(t)]}{c} \sin(\omega_0 t)$$

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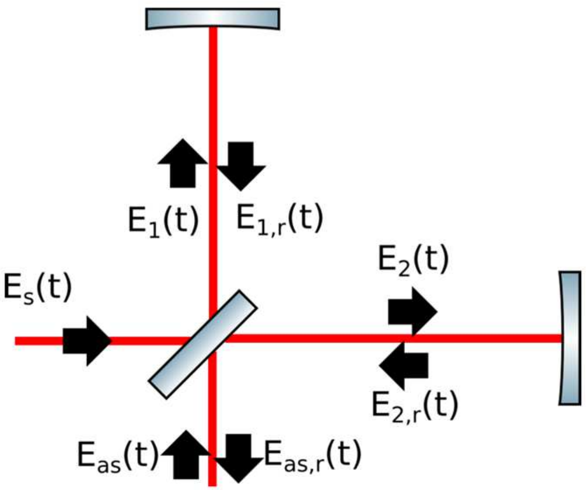
Electric field in simple Michelson interferometer



$$x_1(t) - x_2(t) = x_{cl,1}(t) - x_{cl,2}(t) + \delta\hat{x}_1(t) - \delta\hat{x}_2(t) + Lh(t)$$

Thermal, seismic — Radiation pressure — GW source

$$h_{SQL} = \sqrt{\frac{4\hbar}{M\Omega^2 L^2}}$$

M = mass of mirror(test mass)
Ω = signal frequency
L = arm length

Squeezed States for Advanced Gravitational Wave Detectors, B.A., University of California Berkeley, Eric Oelker (2009)

Standard quantum limit of GW detector

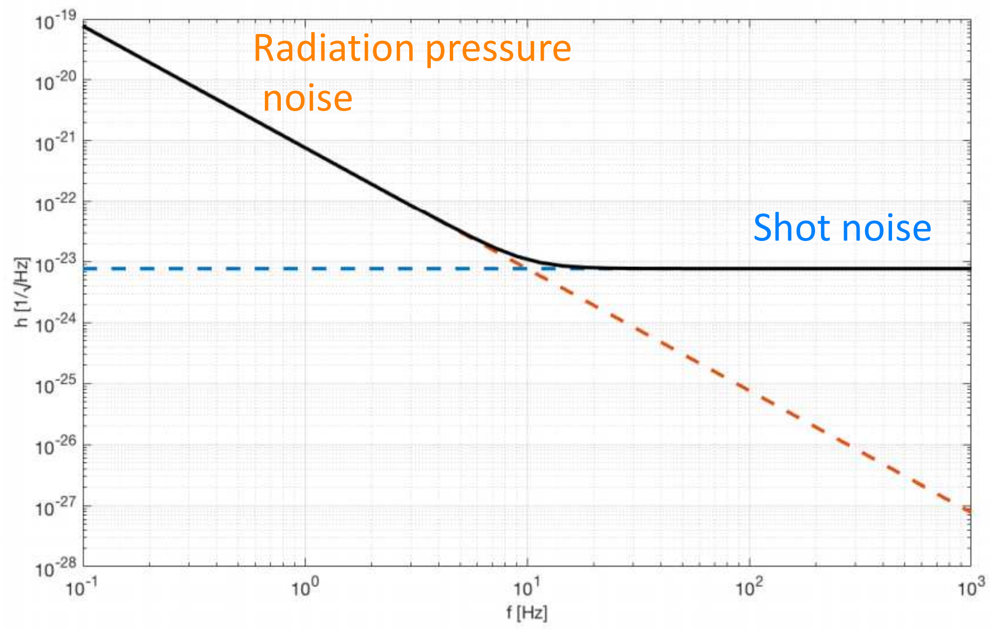
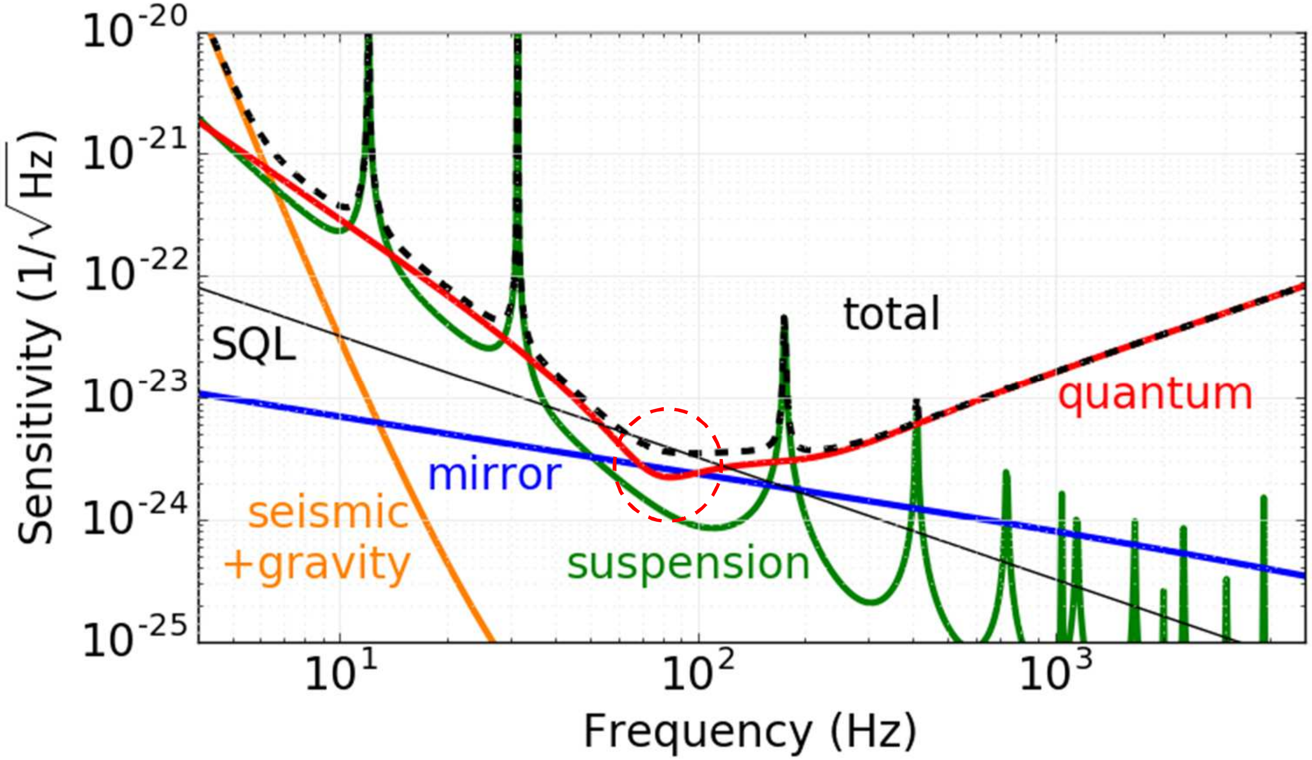


Figure 2.3: The strain equivalent quantum noise is plotted for an interferometer with M = 50 kg, L = 3 km, P = 10 MW. The two contribution of radiation pressure noise and shot noise are shown in blue and red respectively.

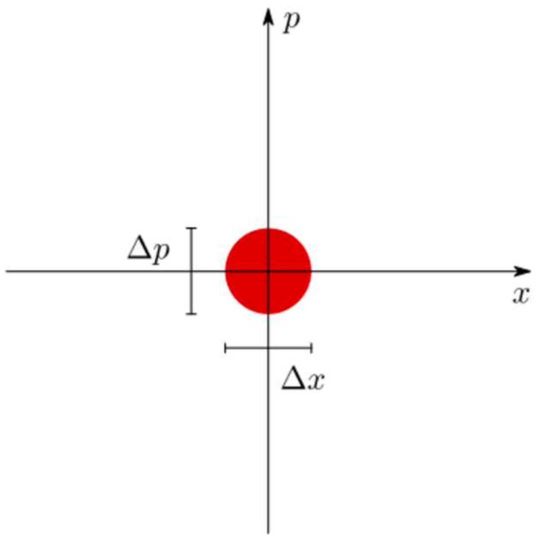
Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

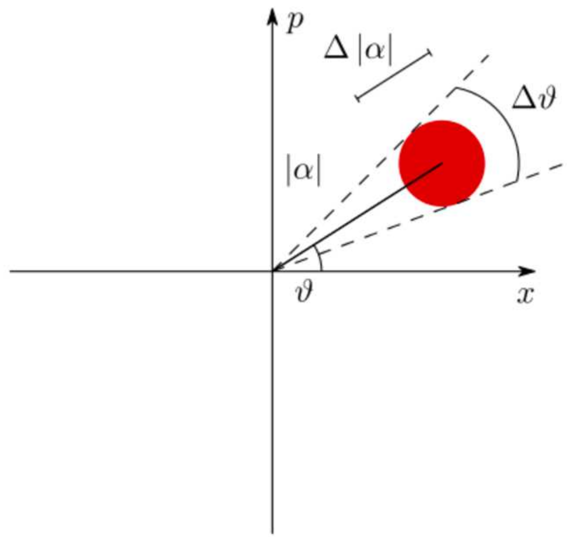
Target sensitivity of KAGRA



Squeezed vacuum injection in GW detector

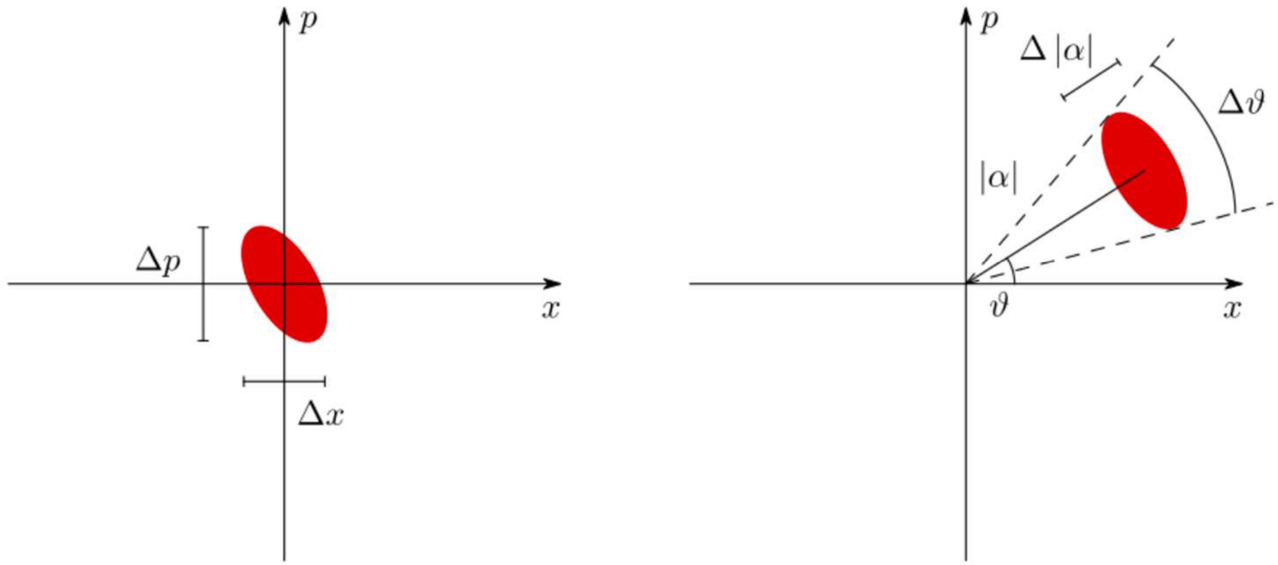
Quadrature in coherent state





Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Quadrature in squeezed state



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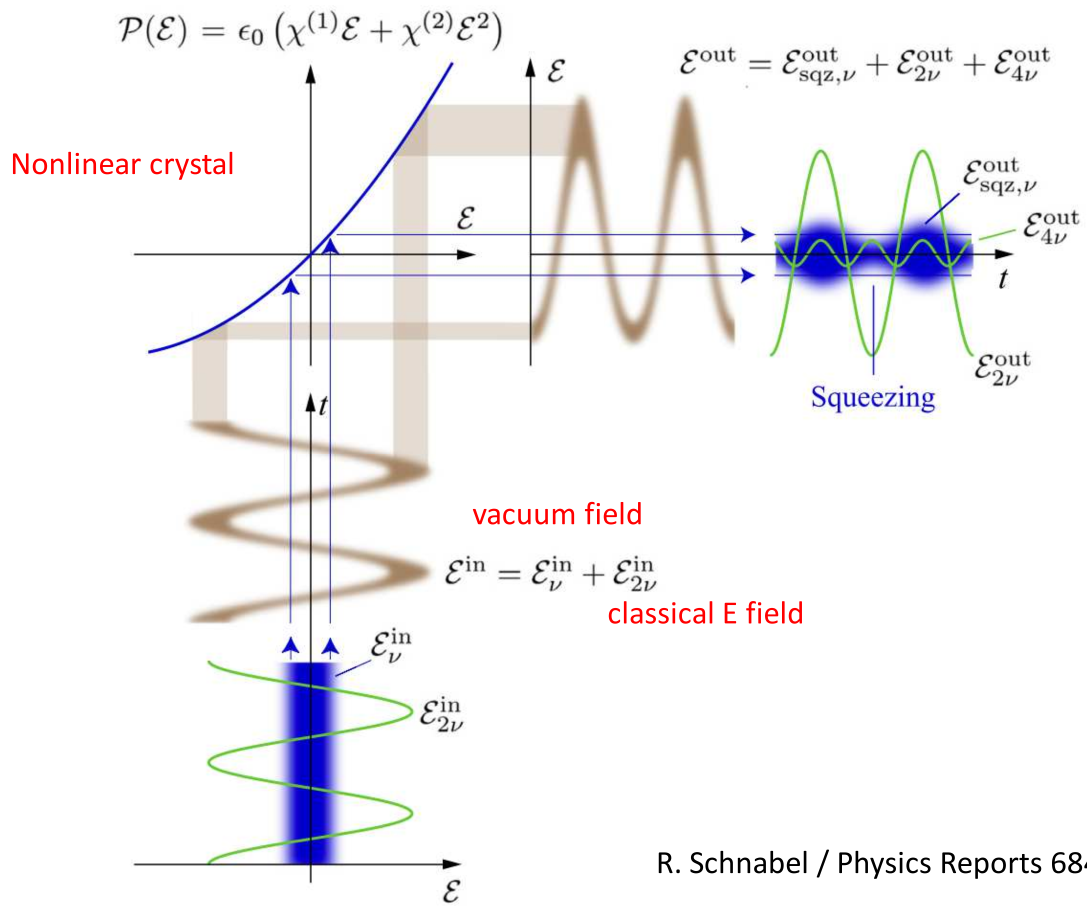
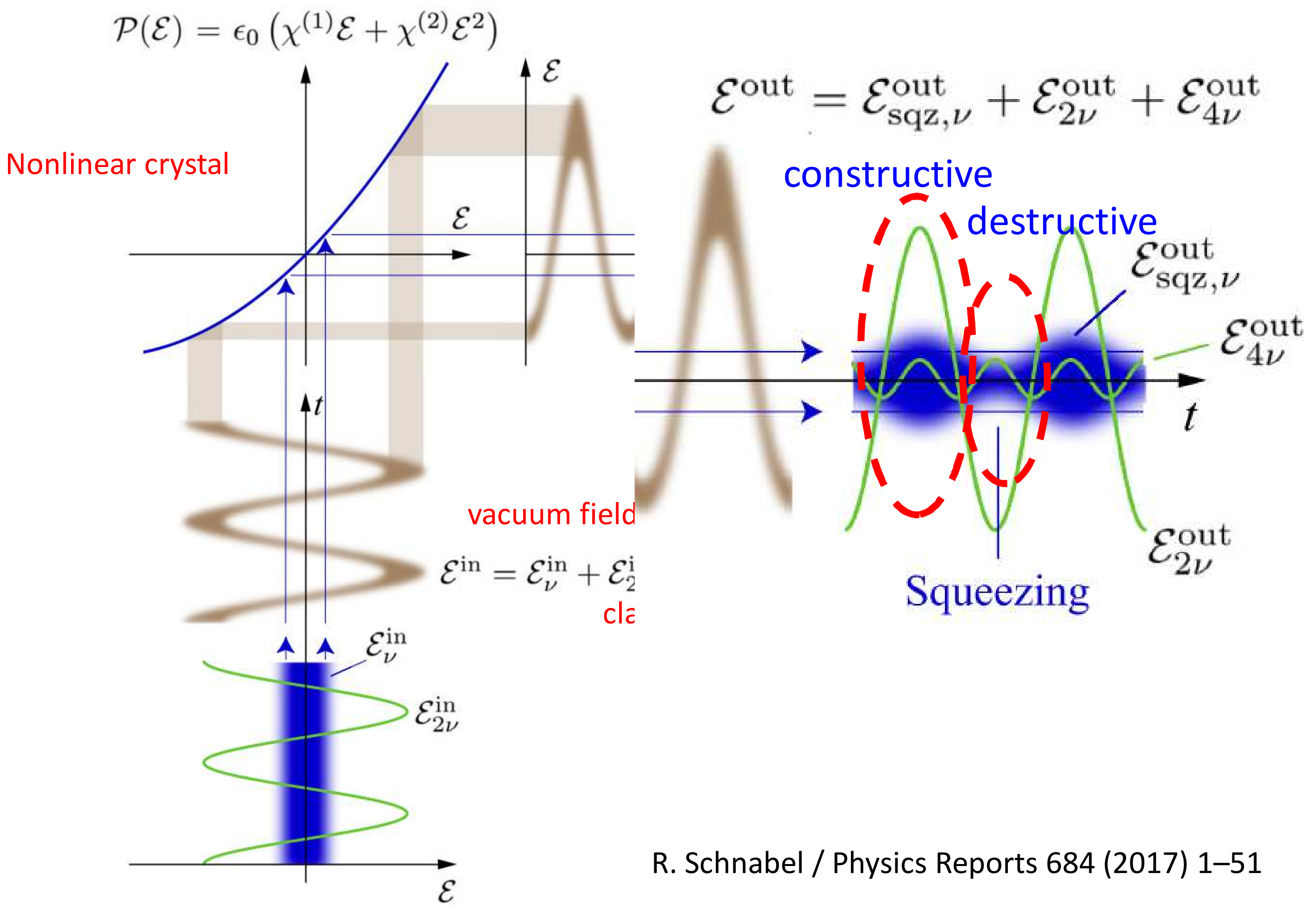
Parametric down conversion process



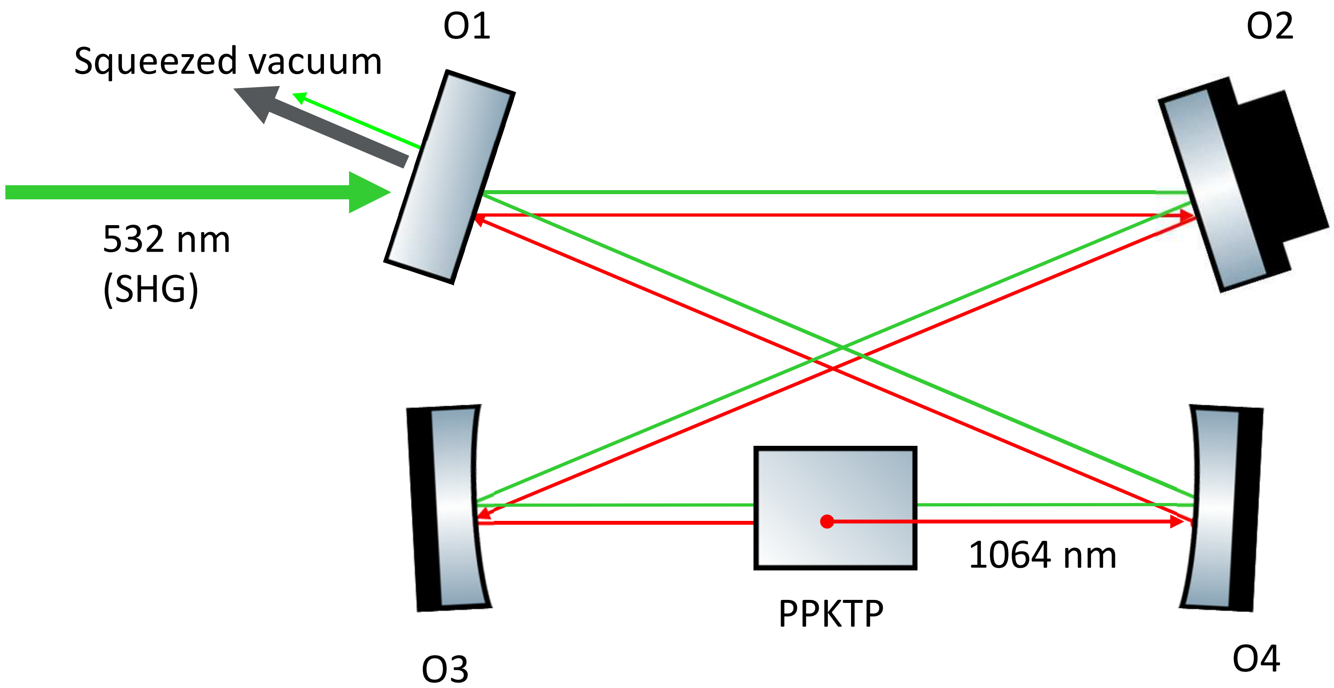
R. Schnabel / Physics Reports 684 (2017) 1–51

Parametric down conversion process



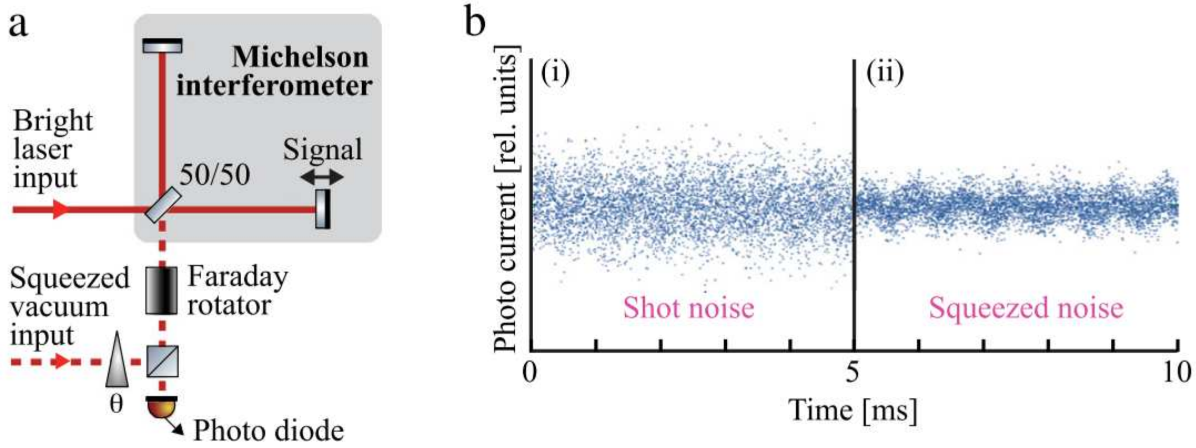
R. Schnabel / Physics Reports 684 (2017) 1–51

Parametric down conversion



Under threshold operation

Squeezed state of light



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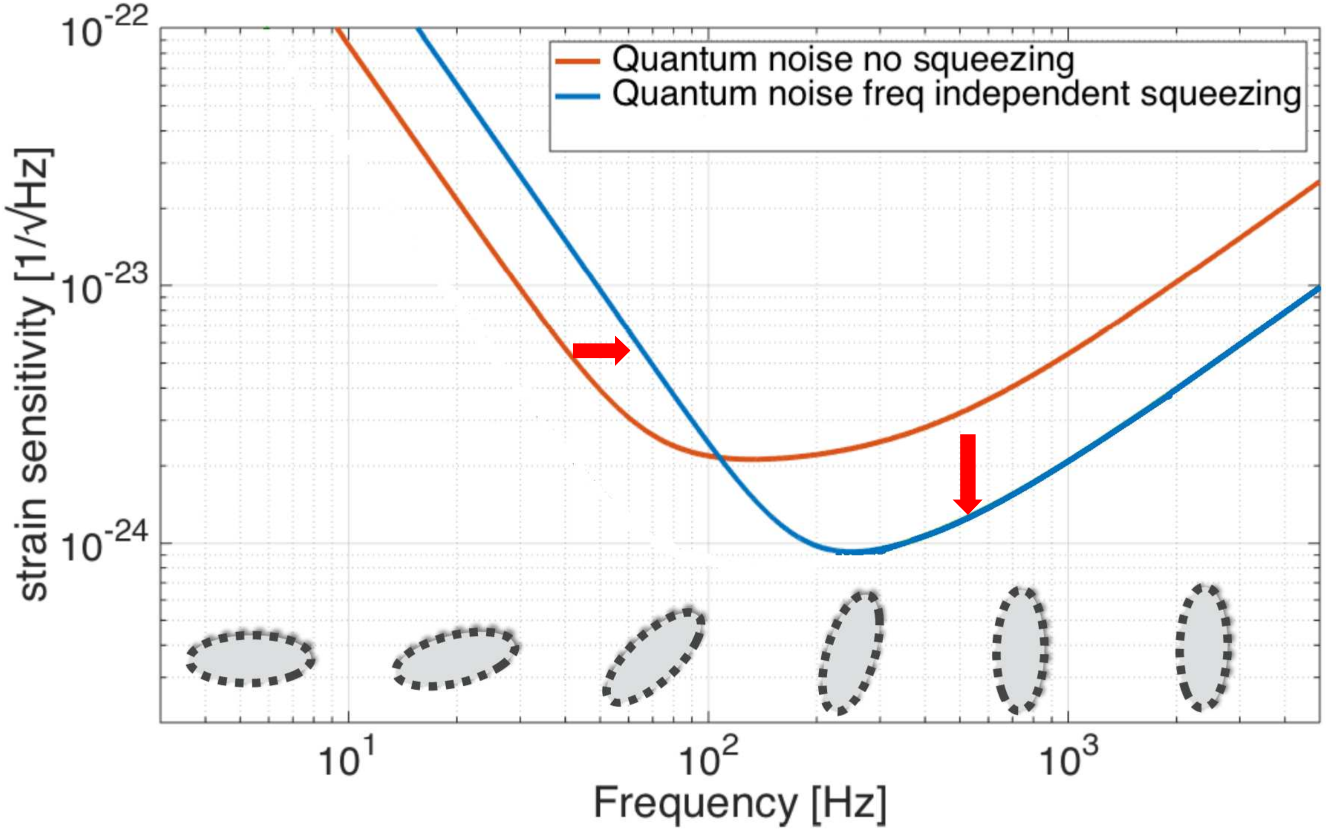
Frequency independent squeezing



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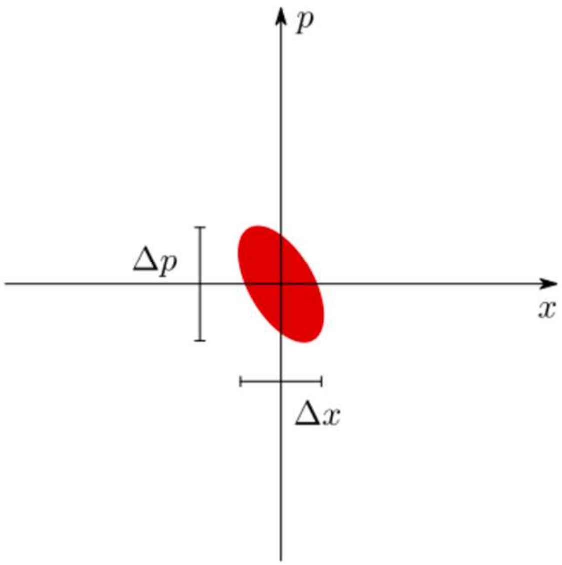
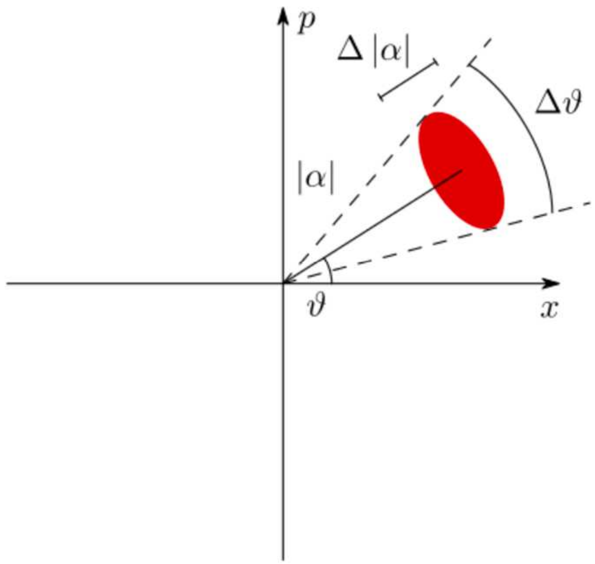
Quadrature in squeezed state





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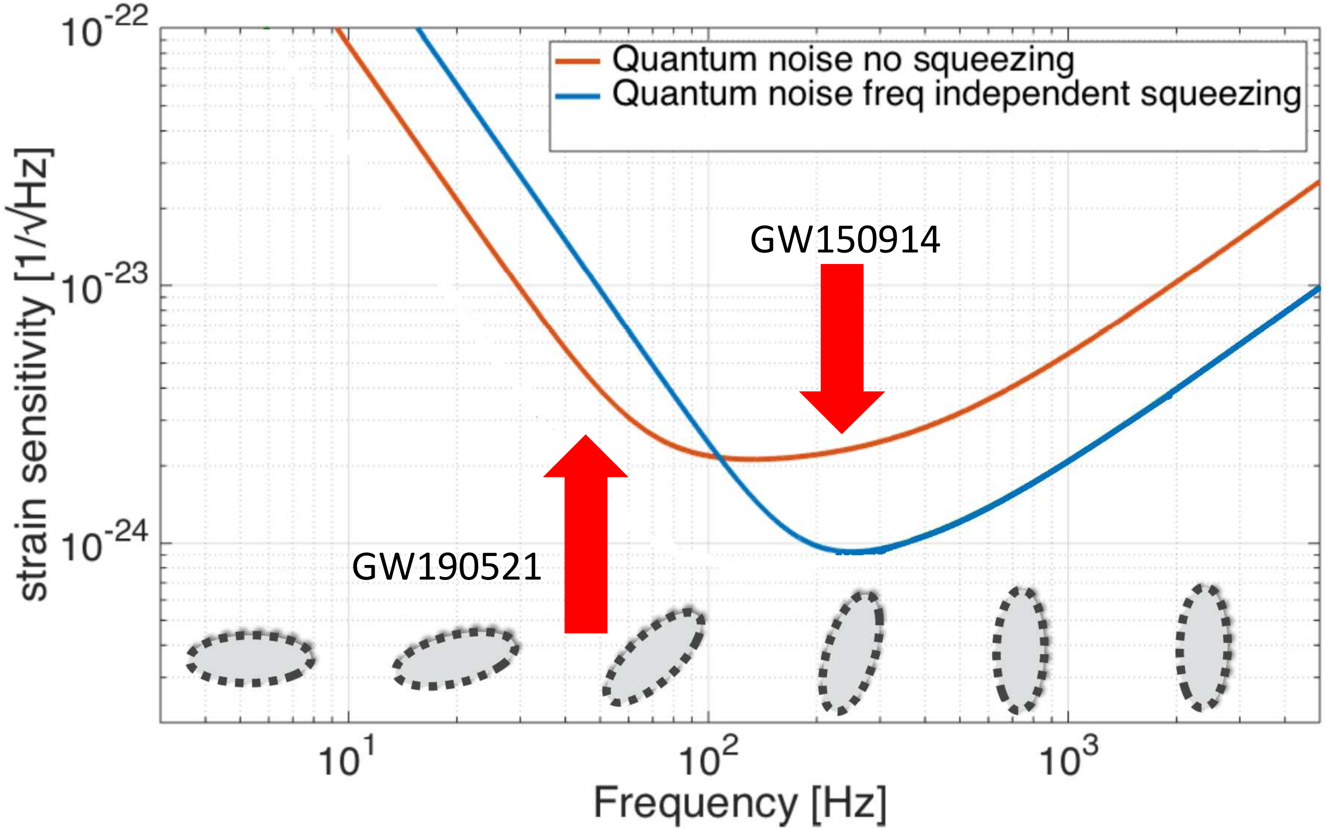
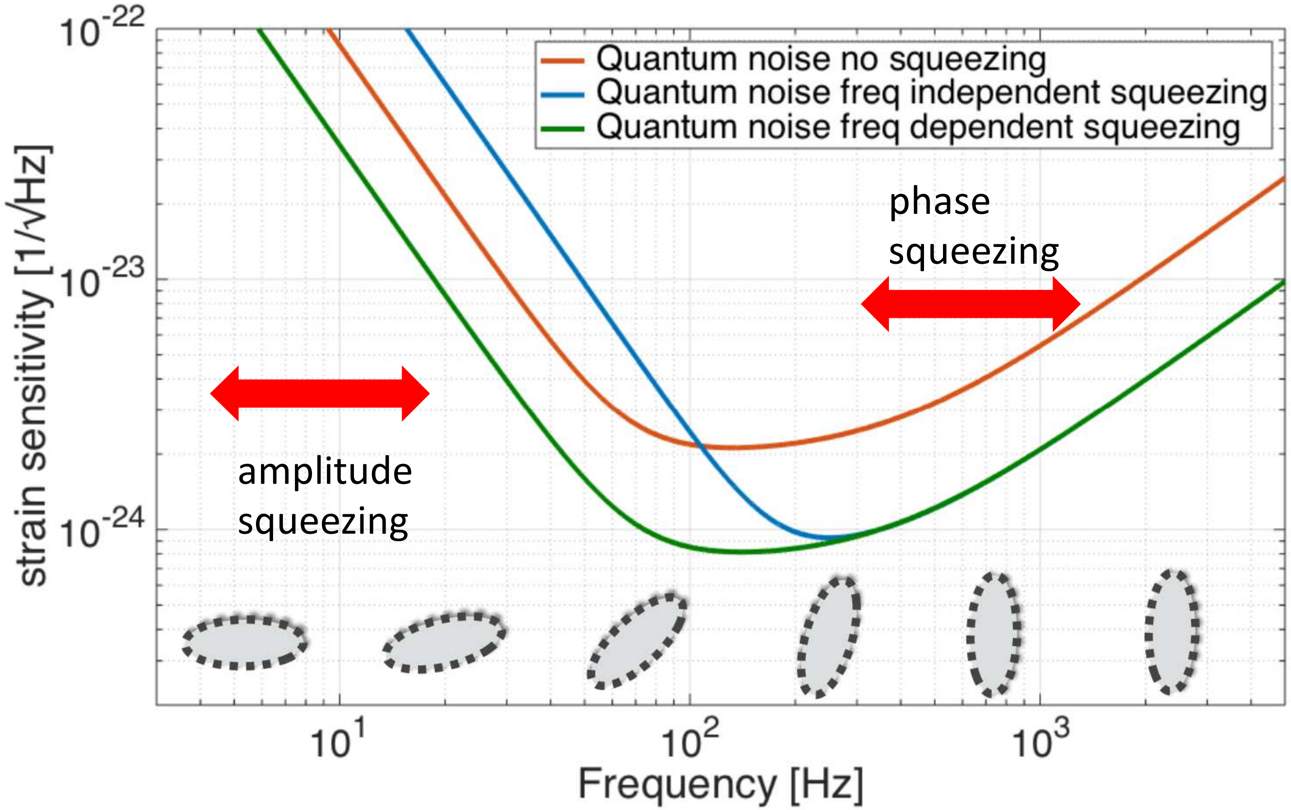
Frequency independent squeezing



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Frequency dependent squeezing(FDS)



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Frequency dependent squeezing in GW detector

First suggestion of filter cavity in FD squeezing

Conversion of conventional gravitational-wave interferometers into quantum nondemolition interferometers by modifying their input and/or output optics

H. J. Kimble,[1] Yuri Levin,[2,*] Andrey B. Matsko,[3] Kip S. Thorne,[2] and Sergey P. Vyatchanin[4]
[1]*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125*
[2]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125*
[3]*Department of Physics, Texas A&M University, College Station, Texas 77843-4242*
[4]*Physics Faculty, Moscow State University, Moscow, 119899, Russia*

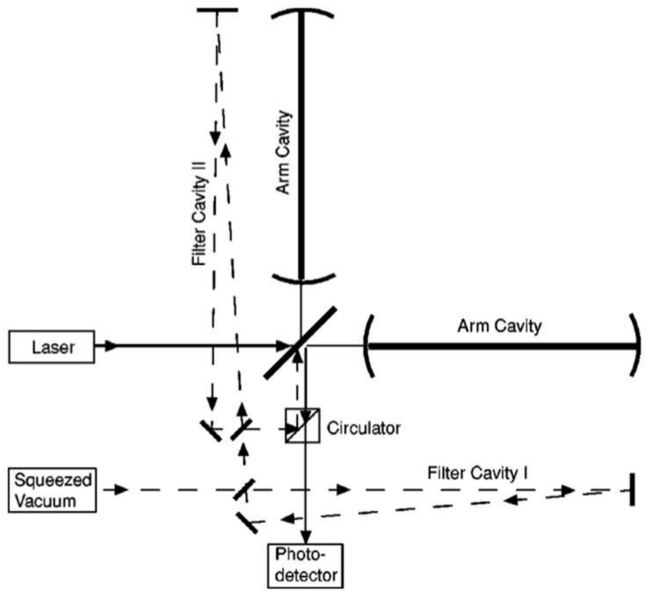
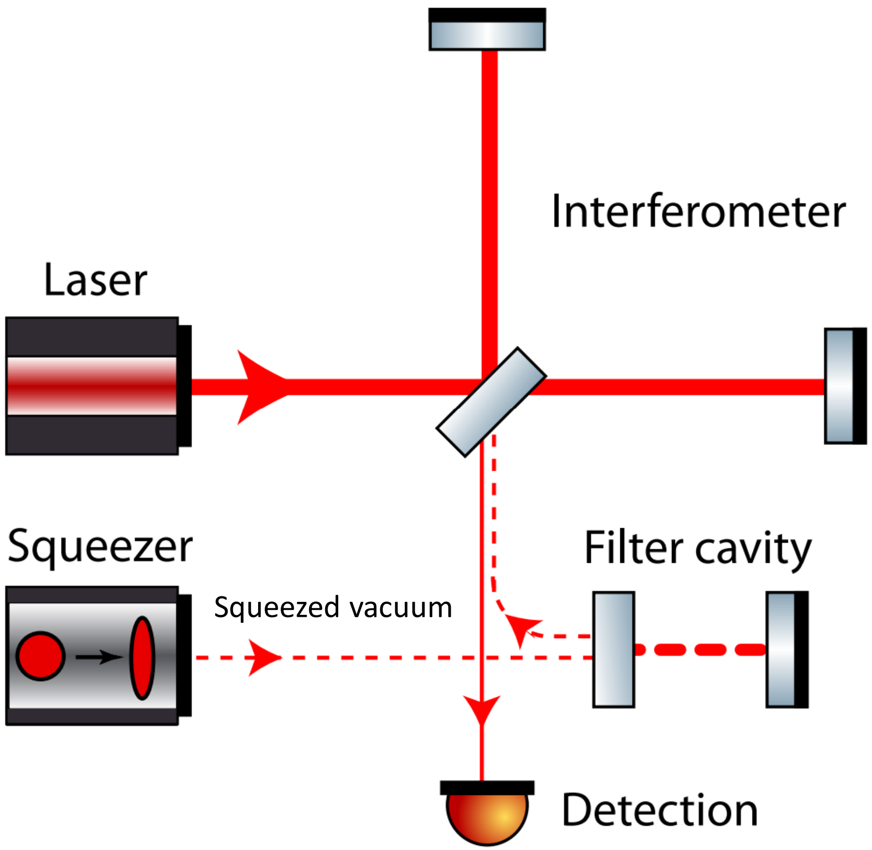


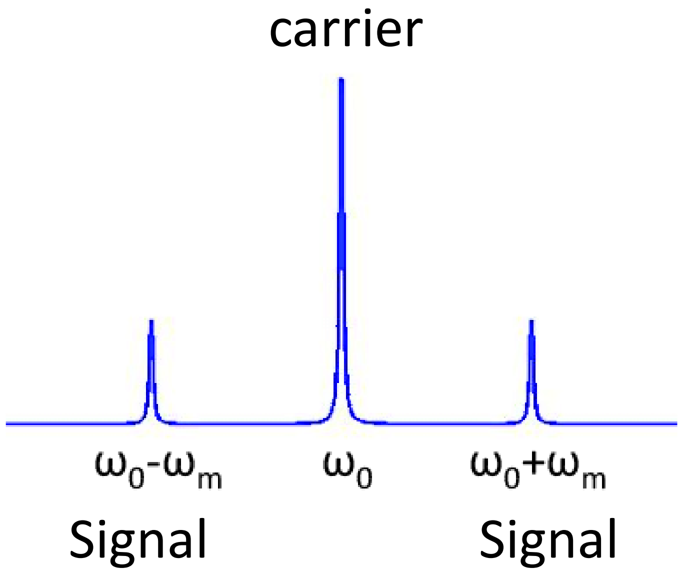
FIG. 1. Schematic diagram of a squeezed-input interferometer.

Squeezed vacuum injection with filter cavity



M. Evans, L. Barsotti, P. Kwee, J. Harms, and H. Miao
Phys. Rev. D **88**, 022002 – Published 29 July 2013

Side band figure



Side band creation

Sideband creation [edit]

We can illustrate the creation of sidebands with one trigonometric identity:

$$\cos(A) \cdot \cos(B) \equiv \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

Adding $\cos(A)$ to both sides:

$$\cos(A) \cdot [1+\cos(B)] = \frac{1}{2}\cos(A+B) + \cos(A) + \frac{1}{2}\cos(A-B)$$

Substituting (for instance) $A \triangleq 1000 \cdot t$ and $B \triangleq 100 \cdot t,$ where t represents time:

$$\underbrace{\cos(1000\,t)}_{\text{carrier wave}} \cdot \underbrace{[1+\cos(100\,t)]}_{\text{amplitude modulation}} = \underbrace{\frac{1}{2}\cos(1100\,t)}_{\text{upper sideband}} + \underbrace{\cos(1000\,t)}_{\text{carrier wave}} + \underbrace{\frac{1}{2}\cos(900\,t)}_{\text{lower sideband}}.$$

Detuned cavity

Stefan Hild et al, “Detuned arm cavities”, 3rd GEO simulation workshop, Hannover, June 2007

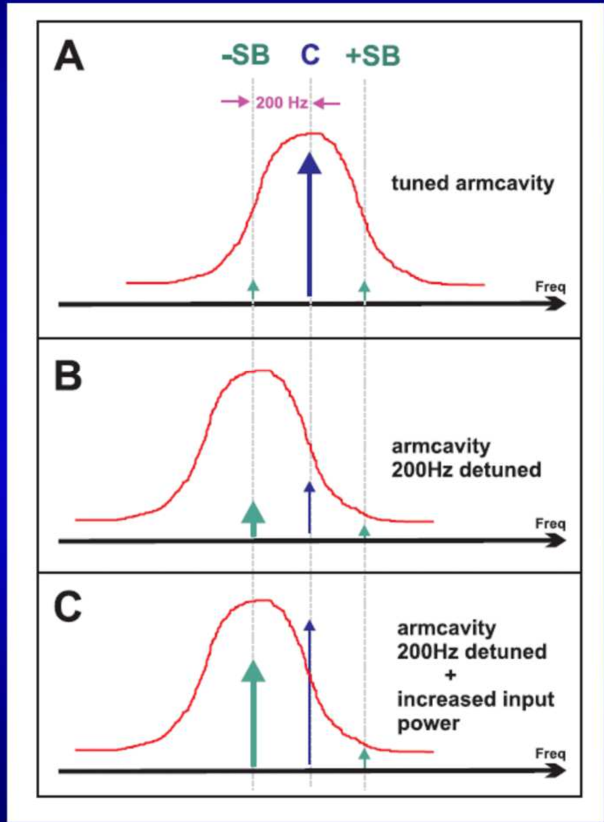
Simple picture

B:
- less carrier light in cavity => less GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced.

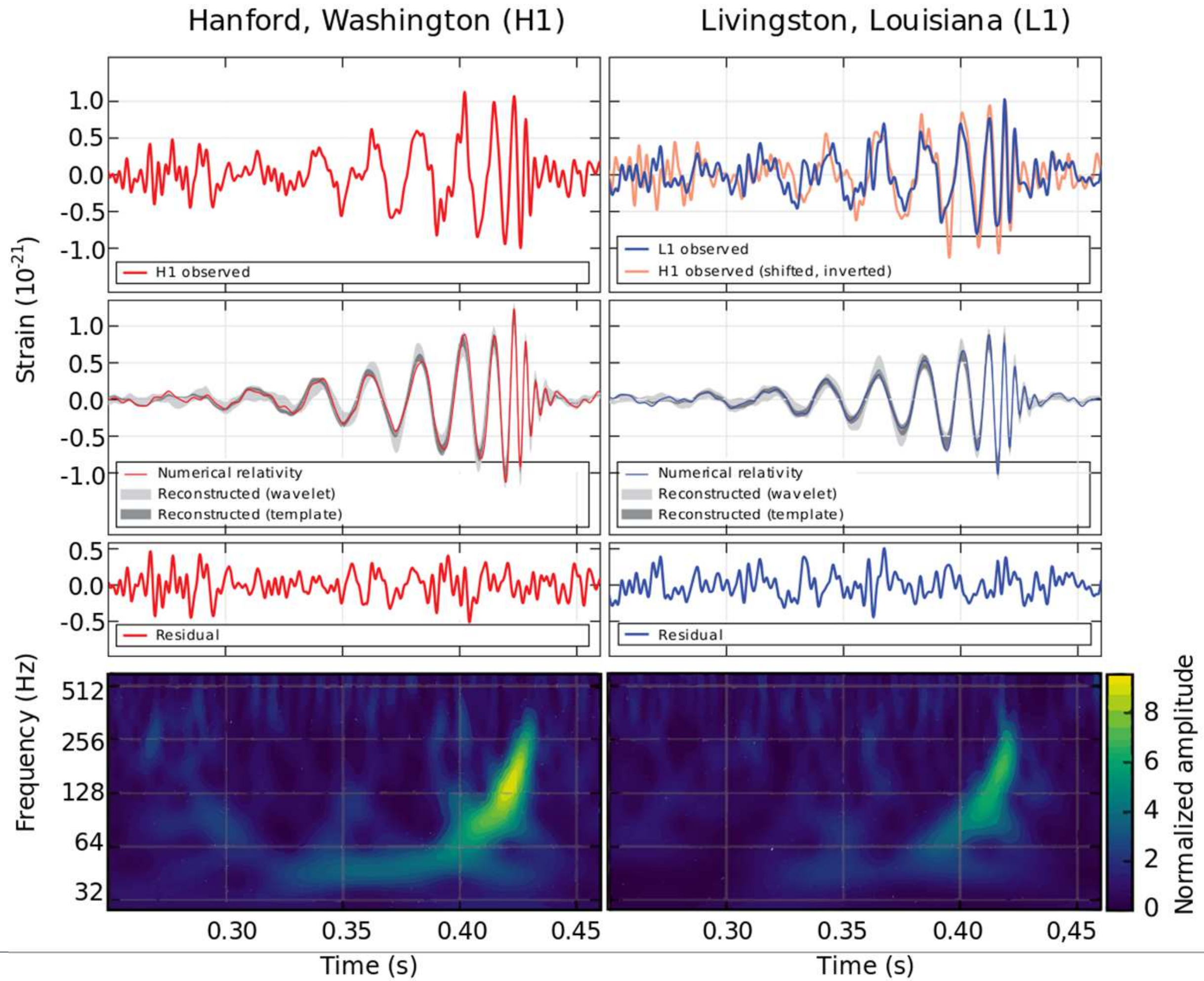
=> Smaller GW signal

C:
- optical power is restored in the cavity by larger PR-gain.
- Same amount of GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced. Overall we win GW signal.

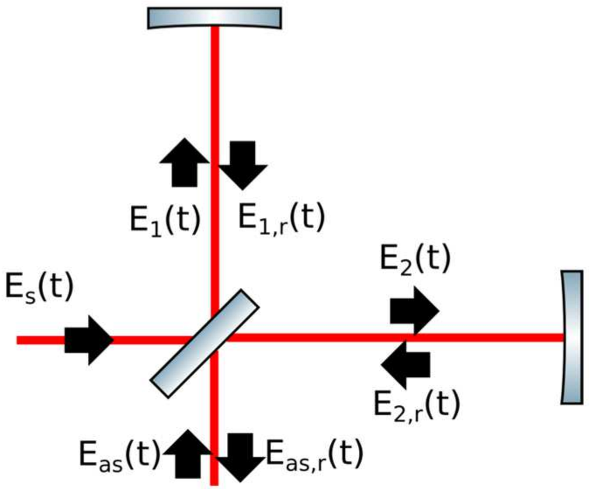
=> Larger GW signal



Gravitational wave signal



Electric field in simple Michelson interferometer



Assume anti-symmetric port is dark port

$$E_1(t) = \frac{1}{\sqrt{2}}[E_s(t) + E_{as}(t)]$$

$$E_2(t) = \frac{1}{\sqrt{2}}[E_s(t) - E_{as}(t)]$$

'as' is vacuum field

Squeezed States for Advanced Gravitational Wave Detectors, B.A., University of California Berkeley, Eric Oelker (2009)

Quantization of electromagnetic field

$$E^{(+)}(t) = e^{-i\omega_0 t} \int_0^{\infty} \left(E(\Omega)e^{-i\Omega t} + E(-\Omega)e^{+i\Omega t}\right) \frac{d\Omega}{2\pi} \tag{1.4}$$

Here, and throughout this thesis, we will assume that $E(\Omega)$ is only appreciable at frequencies where $\Omega \ll \omega_0$. Therefore, I may formally extend the integrals from zero to infinity for ease of notation. We may rewrite 1.1 as:

$$E(t) = e^{-i\omega_0 t} \int_0^{\infty} \left(E(\Omega)e^{-i\Omega t} + E(-\Omega)e^{+i\Omega t}\right) \frac{d\Omega}{2\pi} + h.c. \tag{1.5}$$

Quantization of electromagnetic field

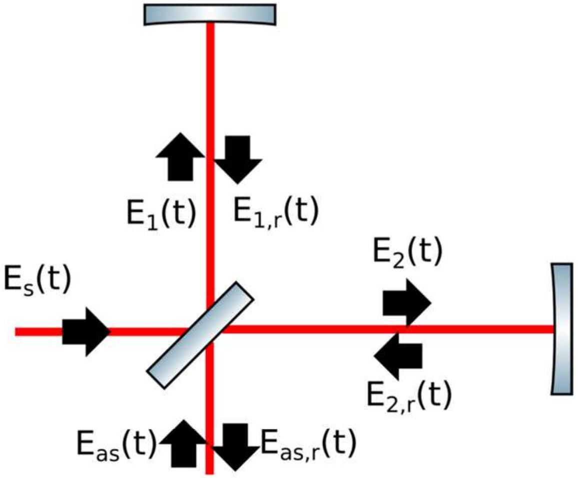
$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\,[a_1(t)\cos(\omega_0 t) + a_2(t)\sin(\omega_0 t)]$$

where we have defined the following quadrature operators:

$$\textbf{Amplitude Quadrature}:\ a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

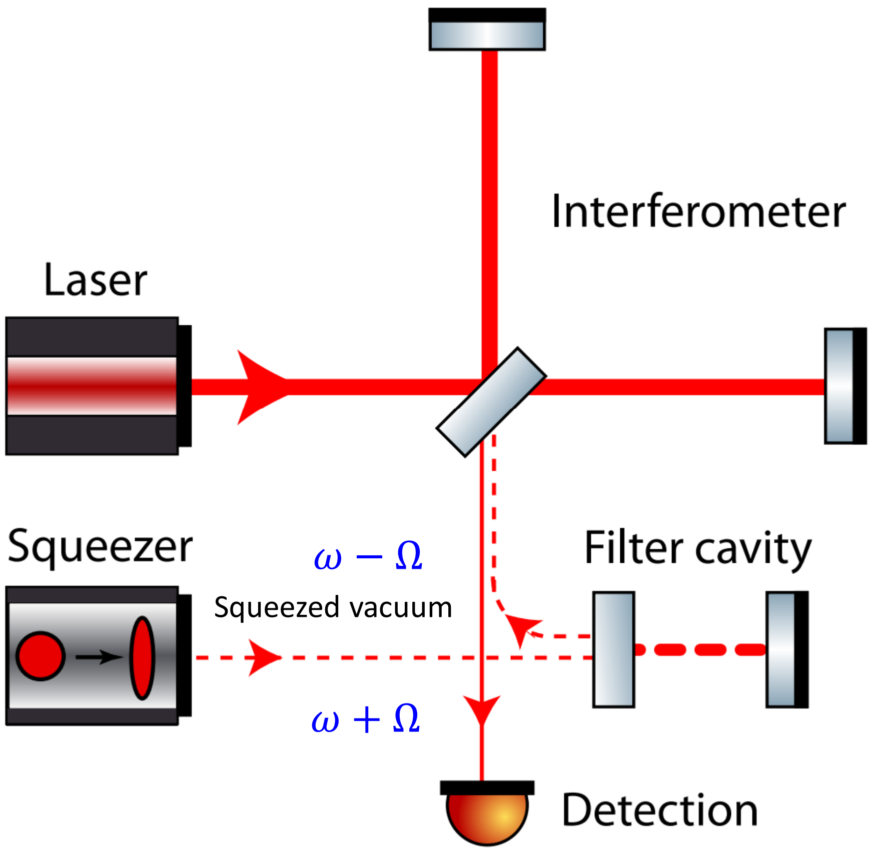
$$\textbf{Phase Quadrature}:\ a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$

Electric field in simple Michelson interferometer



$$E_{as}(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\left[\cos(\omega_0 t)\int_0^\infty \left(a_1(\Omega)e^{-i\Omega t} + a_1^\dagger(\Omega)e^{+i\Omega t}\right)\frac{d\Omega}{2\pi} + \sin(\omega_0 t)\int_0^\infty \left(a_2(\Omega)e^{-i\Omega t} + a_2^\dagger(\Omega)e^{+i\Omega t}\right)\frac{d\Omega}{2\pi}\right]$$

Squeezed vacuum injection with filter cavity



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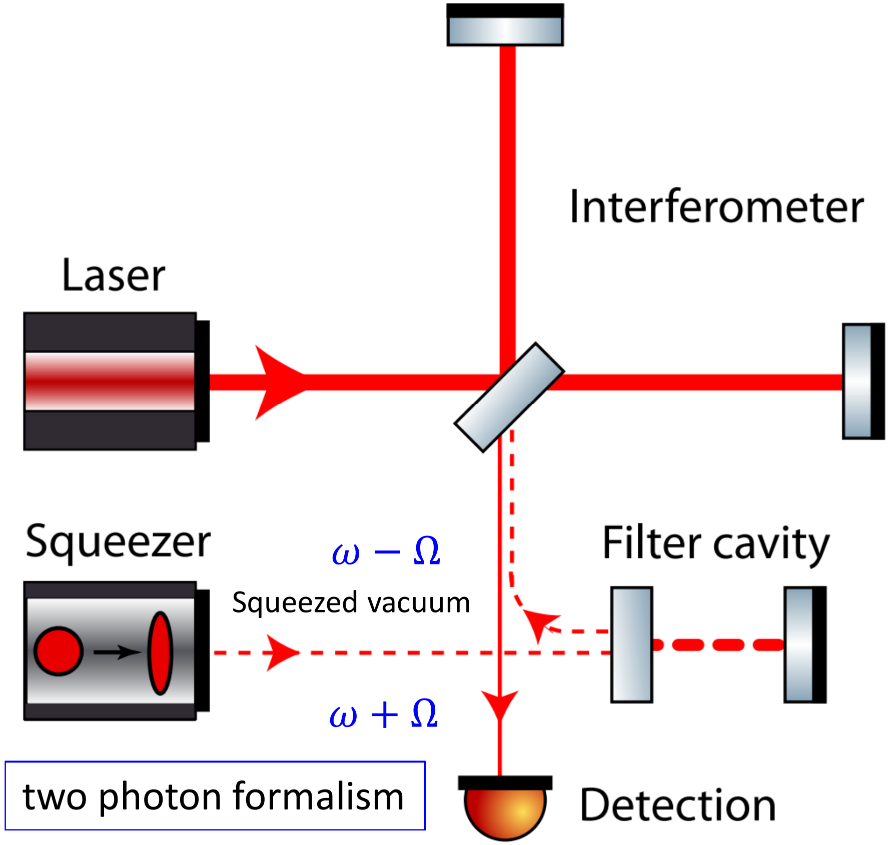
Squeezed vacuum injection with filter cavity



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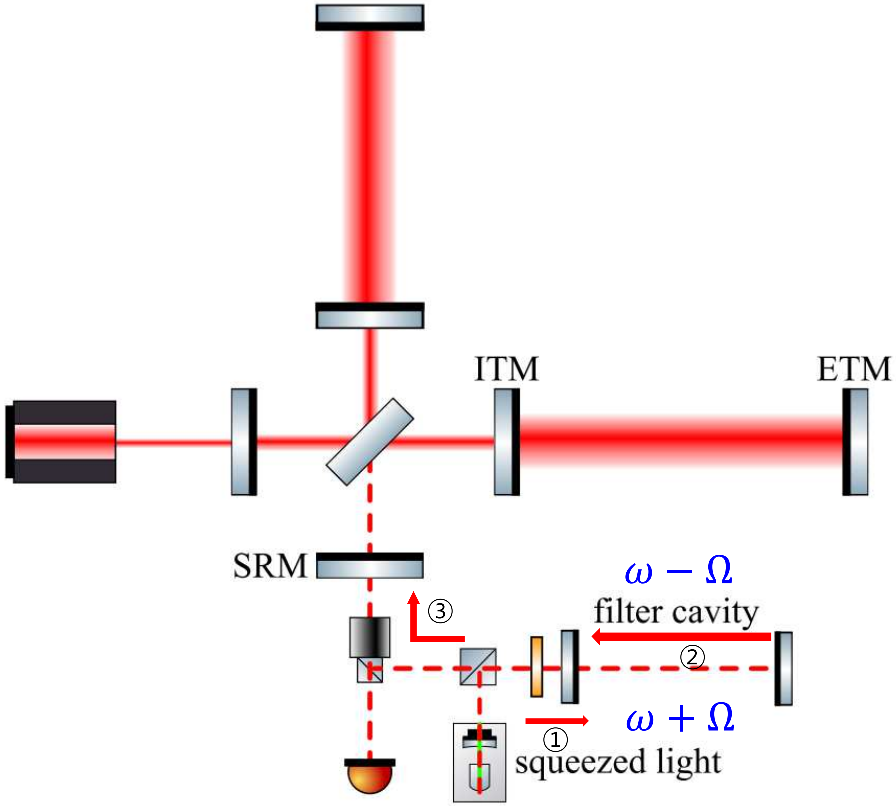
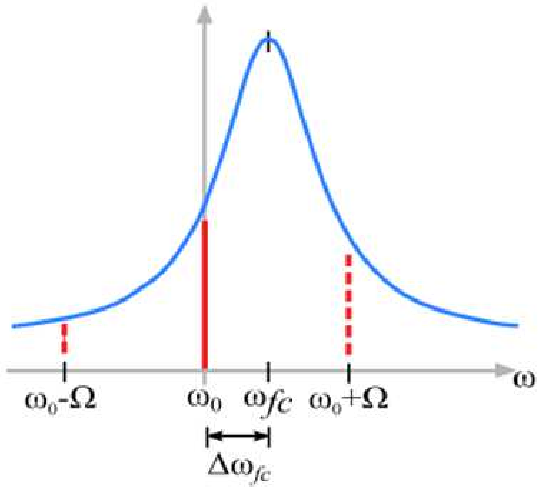
Filter cavity



$$\textbf{Amplitude Quadrature}: \; a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

$$\textbf{Phase Quadrature}: \; a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$



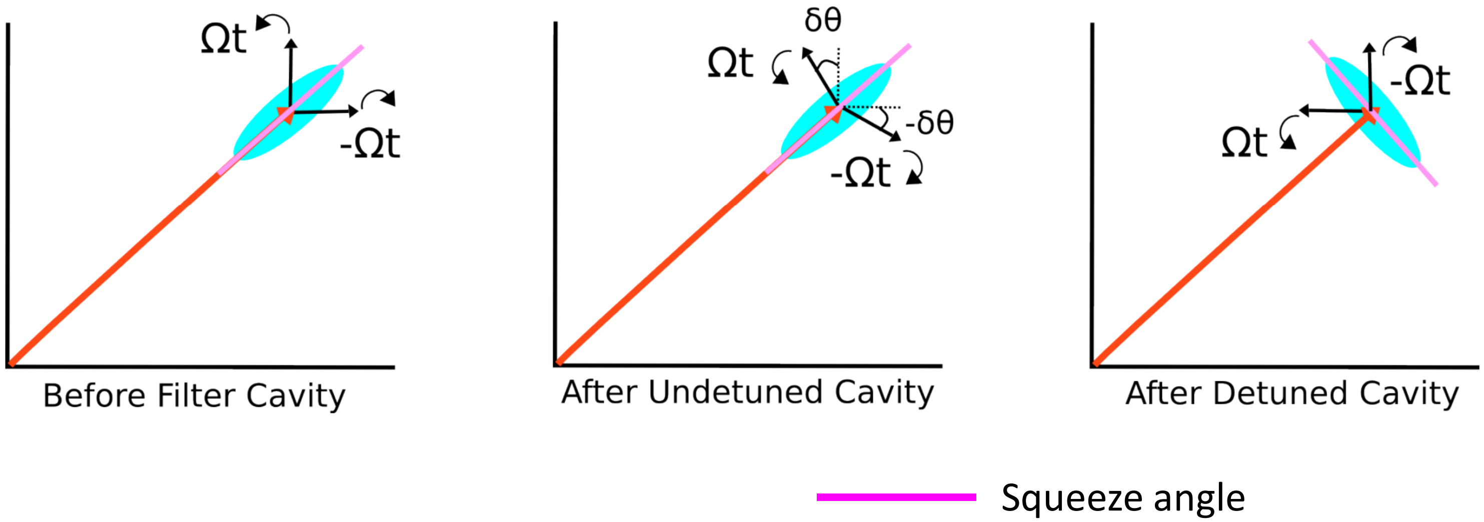
Frequency detuning

Denis Martynov et al, Phys. Rev. D **99**, 102004

Squeeze angle

$$\textbf{Amplitude Quadrature}: \; a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

$$\textbf{Phase Quadrature}: \; a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$



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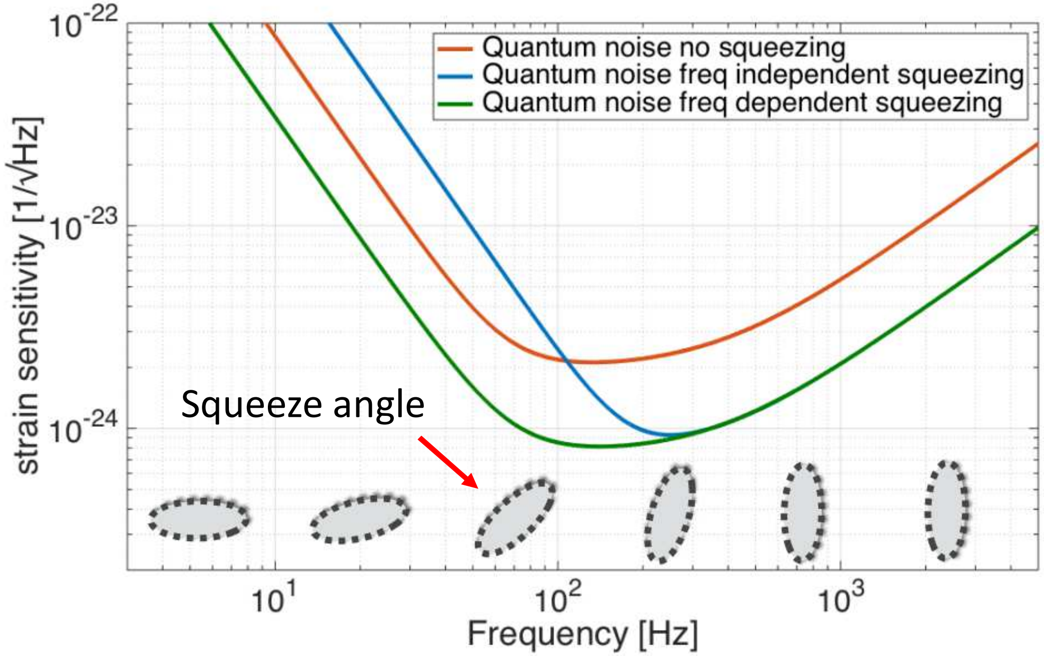
Squeeze angle



Squeeze angle

$$\arctan\left(\frac{2\gamma_{\text{fc}}\Delta\omega_{\text{fc}}}{\gamma_{\text{fc}}^2 - \Delta\omega_{\text{fc}}^2 + \Omega^2}\right)$$

Detuning frequency
< Ω

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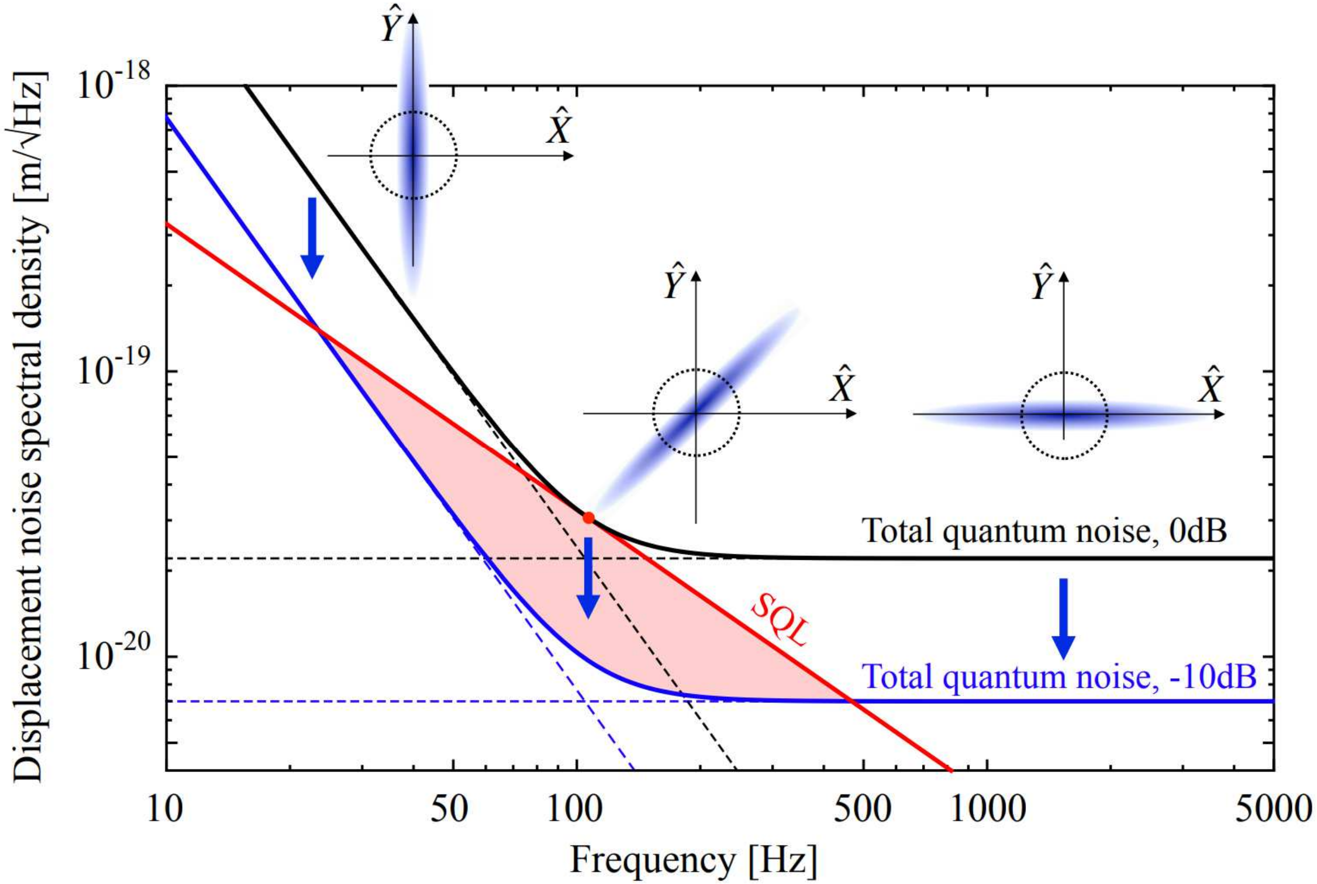
Squeeze angle rotation



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Squeeze angle rotation

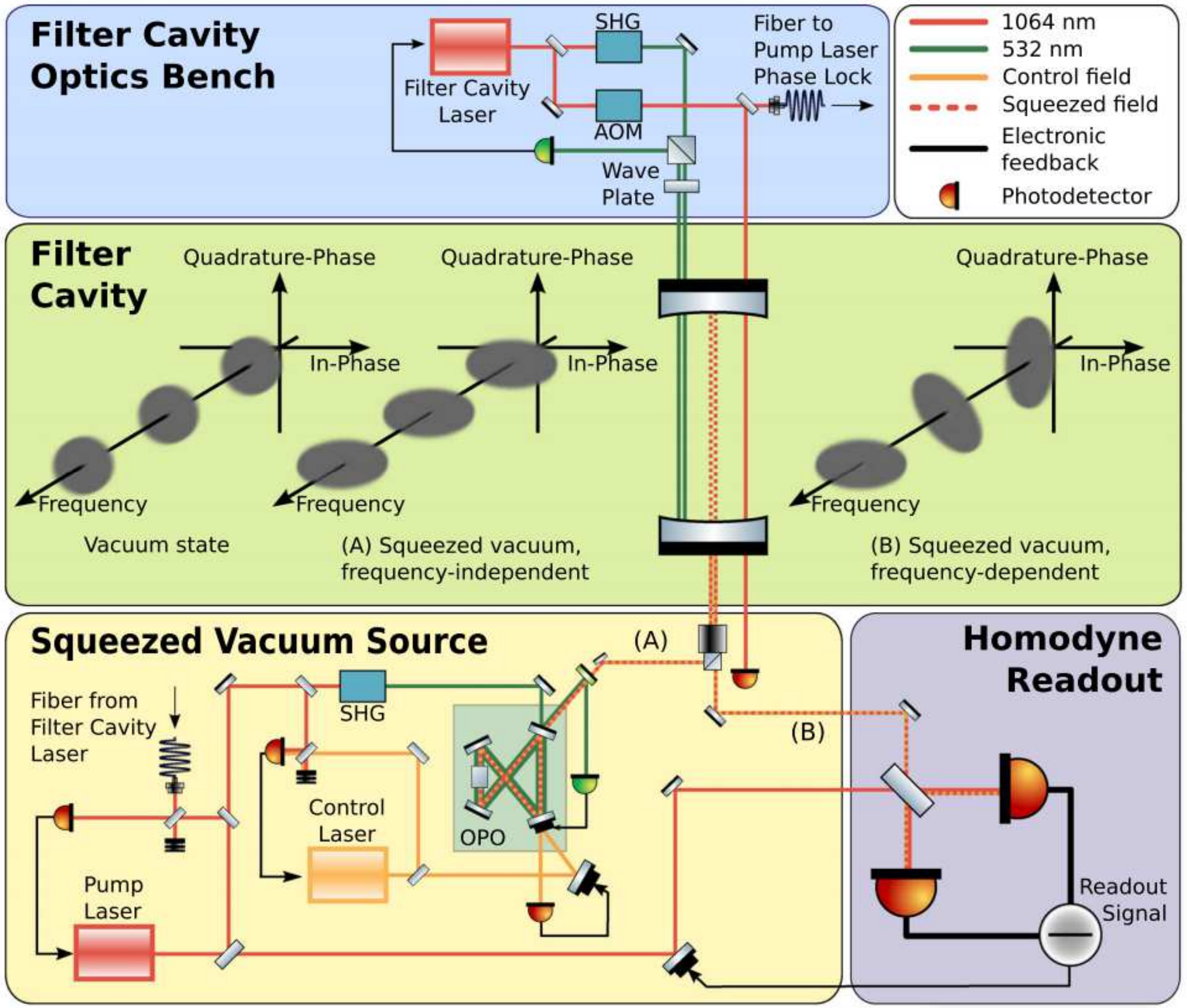
$$\alpha_p = \arctan\left(\frac{2\gamma_{\mathrm{fc}}\Delta\omega_{\mathrm{fc}}}{\gamma_{\mathrm{fc}}^2 - \Delta\omega_{\mathrm{fc}}^2 + \Omega^2}\right)$$

$\gamma = loss\ of\ filter\ cavity$
$\omega_{fc} = detuned\ frequency$

$$t_{\mathrm{st}} = \frac{1}{\gamma_{\mathrm{fc}}} = \frac{\sqrt{2}}{\Omega_{\mathrm{SQL}}} \simeq 3\,\mathrm{ms}$$

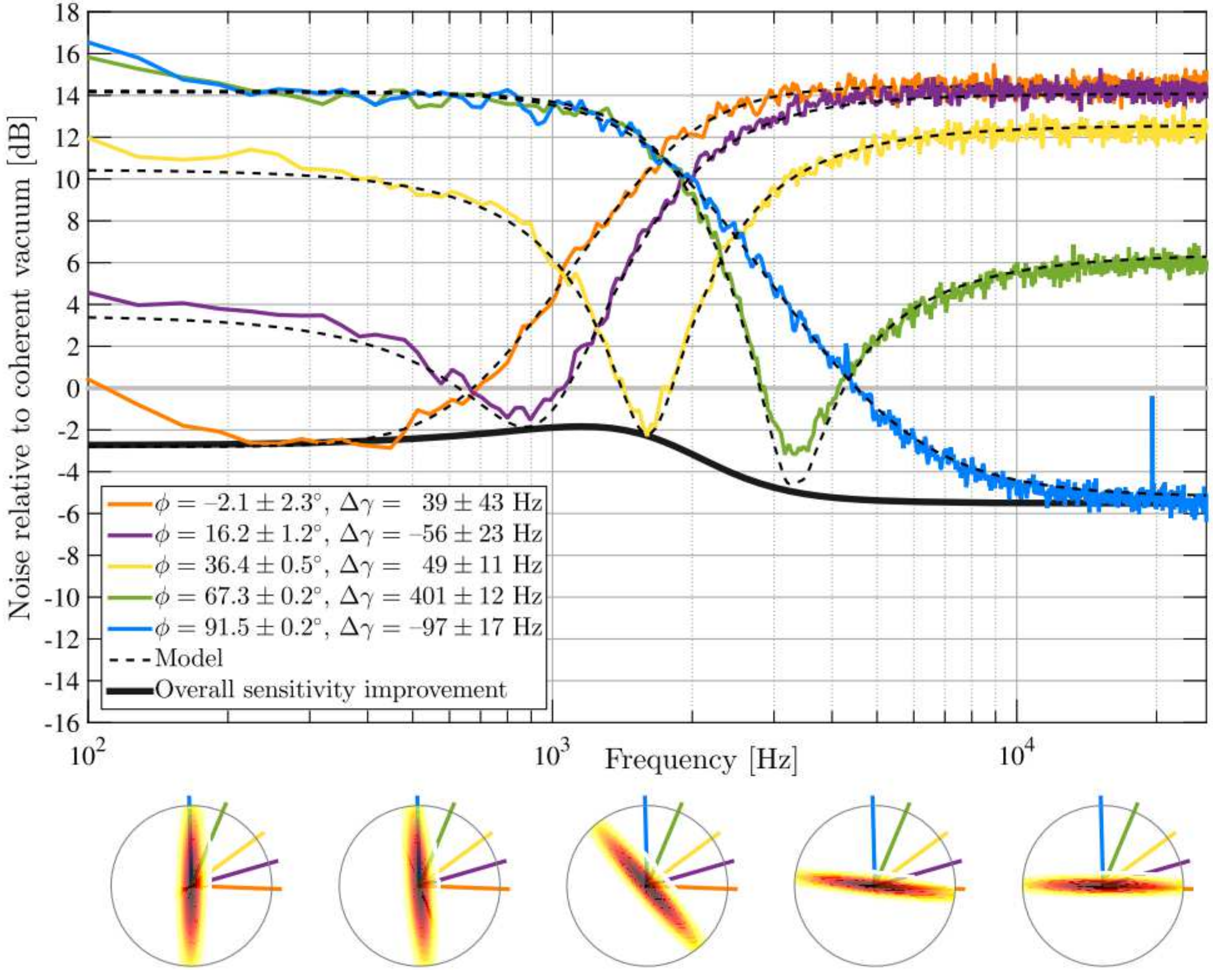
LIGO filter cavity

PRL **116,** 041102 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016



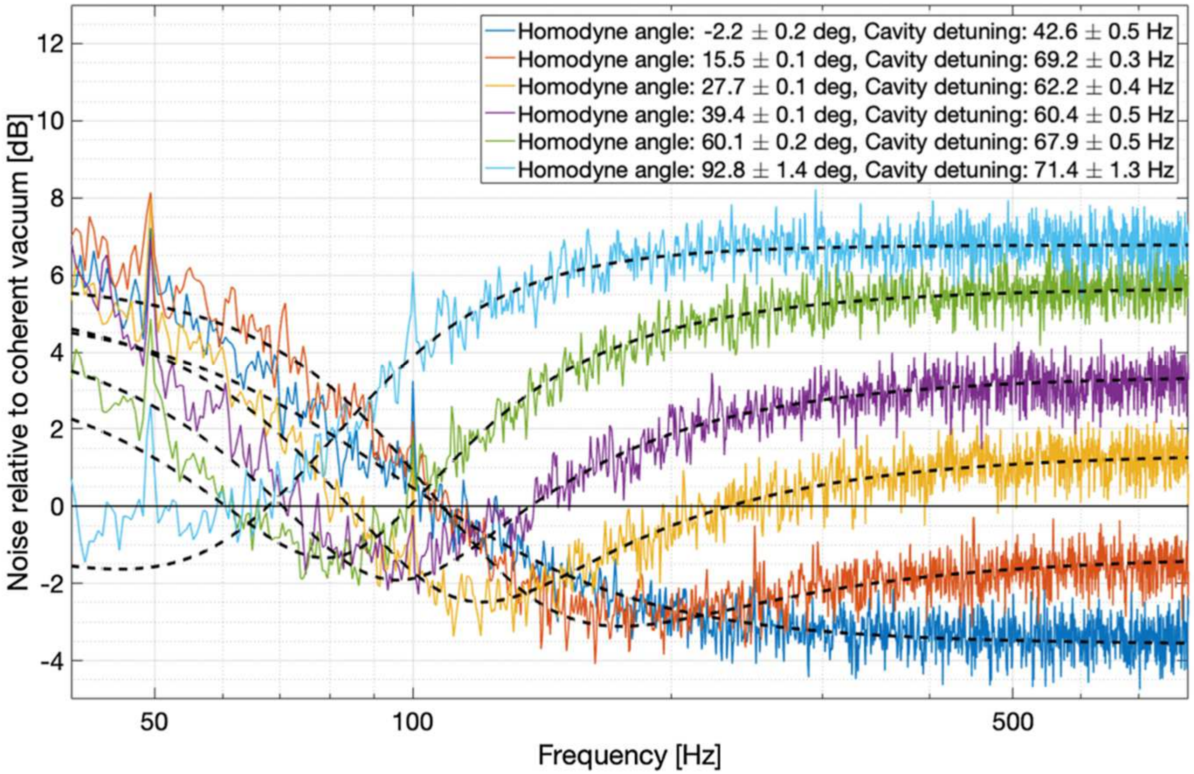
LIGO filter cavity

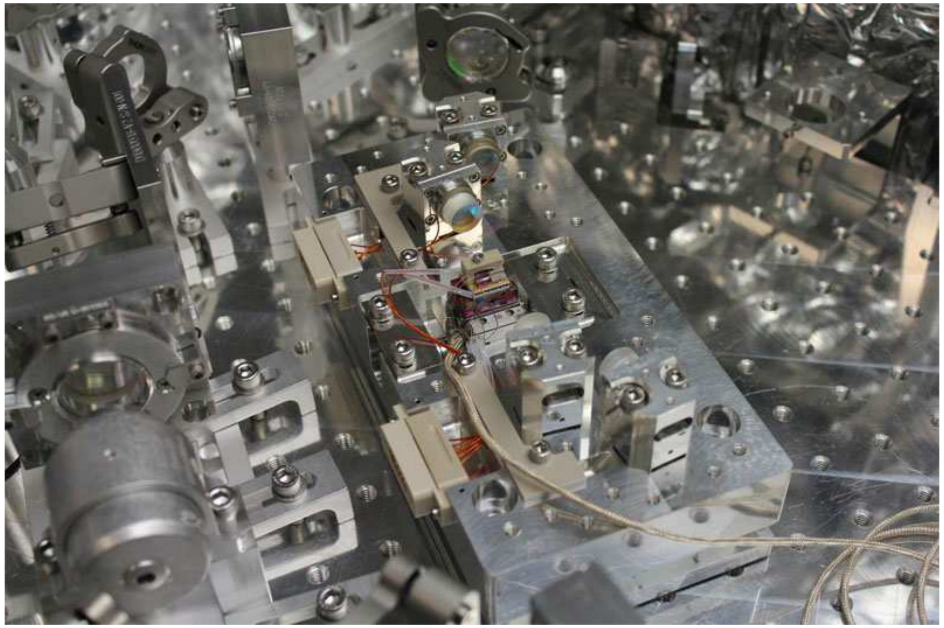
PRL **116,** 041102 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

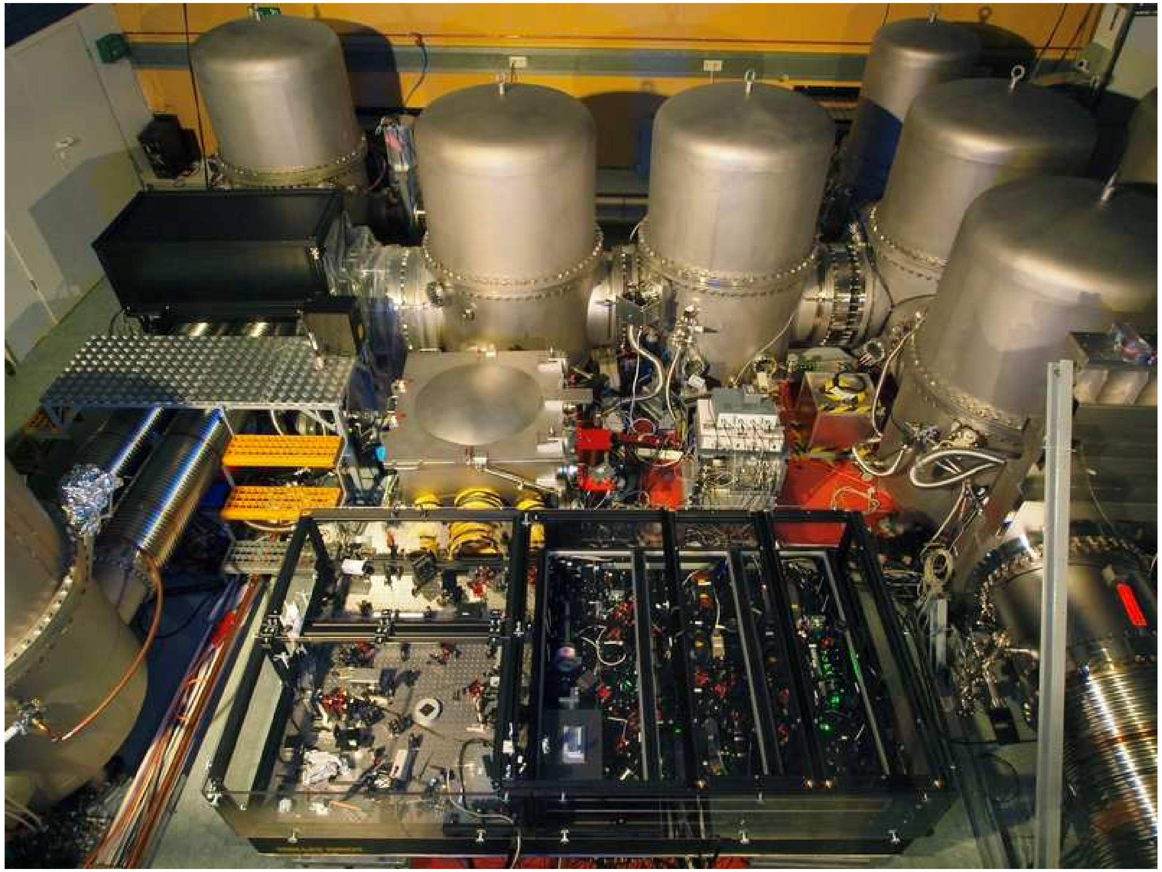


KAGRA filter cavity

PHYSICAL REVIEW LETTERS **124,** 171101 (2020)







Thank you