

# Simulations of Collisions Between Equal-mass Asteroids by the Material Point Method

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# Material Point Method (MPM): Introduction

## What is MPM?

- Hybrid method (Lagrangian + Eulerian), extended from **Particle-in-Cell (PIC)**.
- Increasingly used in recent years for simulating **elastic bodies and granular materials**.
- Widely applied in:
  - **Graphics**: snow, sand, splashes
  - **Space science**: meteor impacts and crater formation
- Primarily designed for solids, but its **grid-based space integration** makes it well-suited for coupling with grid-based phenomena like **fluids** and **heat**.
- Offers significant advantages for **GPU optimization** since particles communicate only with grids and they remain at fixed positions.

## Motivation

- Increasingly explored as a particle-based alternative for fluids, granular flows, and solids like metals
- We investigate its applicability to numerical astrophysics.

→ **Our target application: Collisions Between Equal-mass Asteroids**

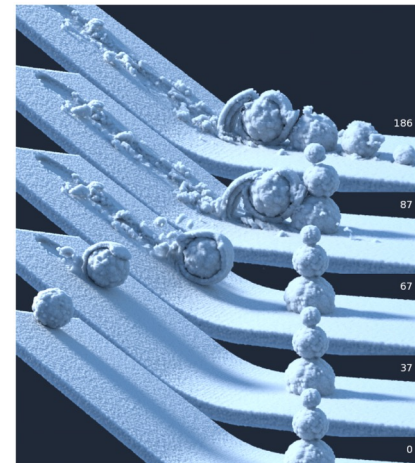
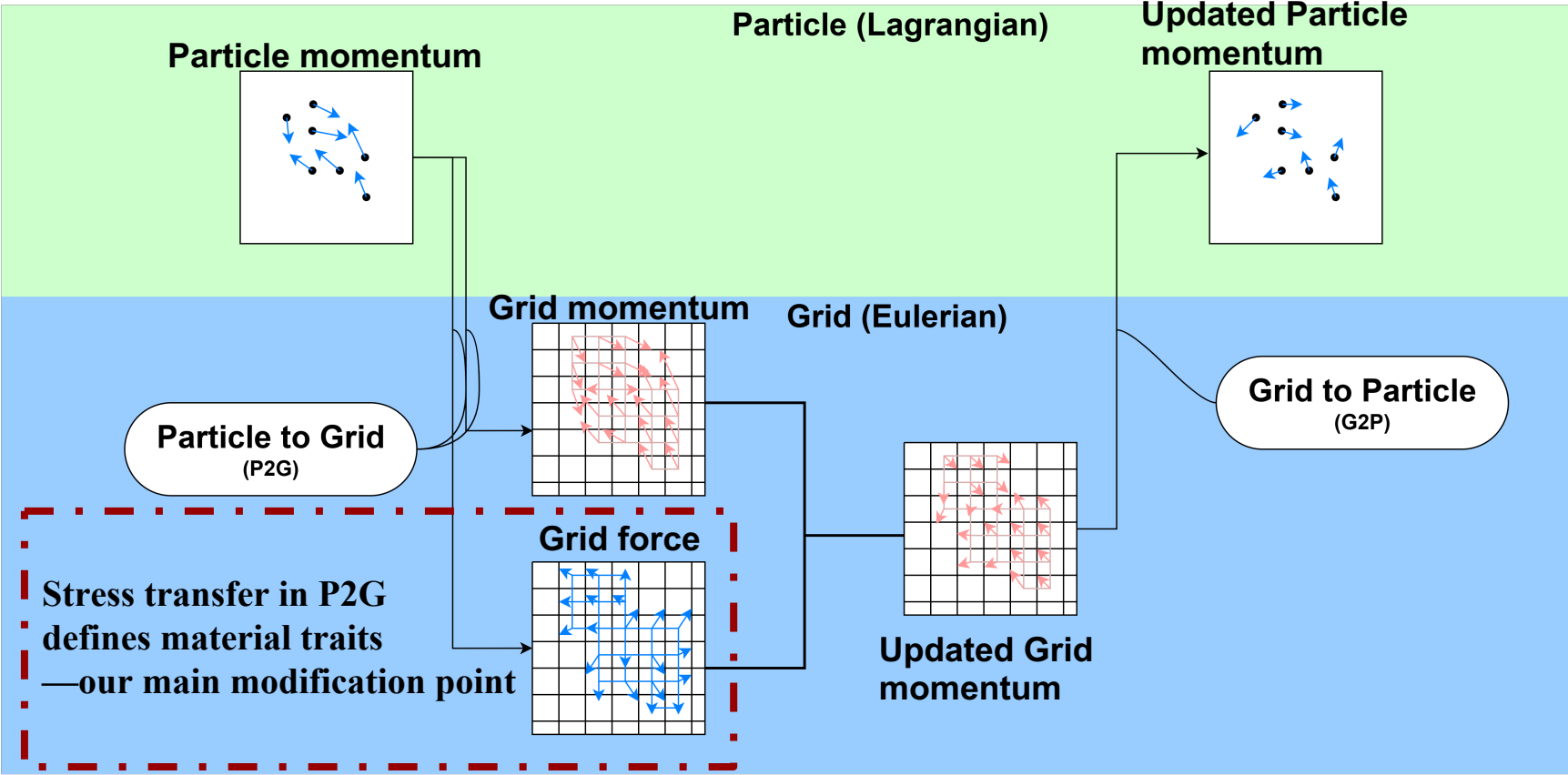


Figure from Stomakhin et al., 2013, ACM Transactions on Graphics

# Material Point Method (MPM): Algorithm flow



# Previous research: SPH-Based Approaches to Asteroid Impact

## Sugiura et al. (2018)

- Motivated by the observation that over half of main-belt asteroids exhibit irregular shapes.
- Simulated asteroid destruction using SPH with a fracture model.
- Asteroid assumed to be homogeneous basalt, with a radius of 50 km.
- Impact velocity and angle varied to analyze remnant shapes.
- Identified impact conditions linked to each shape.

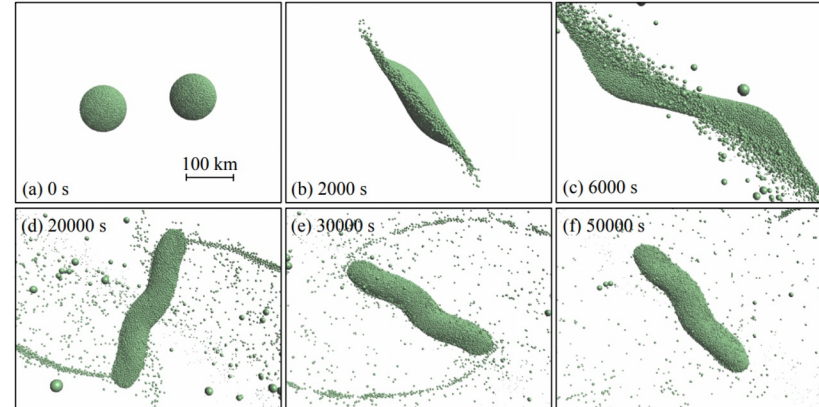


Figure from Sugiura et al., 2018, Astronomy & Astrophysics

# What physical processes are required for asteroid impact?

- ❖ **Fracture distribution:** representing variability in material strength
  - Weibull distribution
- ❖ **Fracture evolution:** modeling crack activation in brittle rock
  - Damage mechanics
- ❖ **Pressure response:** modeling compression using volume change, energy density, and damage
  - Tillotson Equation of State
- ❖ **Elastic deformation:** describing stress response in intact regions
  - Neo-Hookean model
- ❖ **Plastic flow in damaged particles:** simulating granular behavior
  - Drucker-Prager model
- ❖ **Gravitational reaccumulation:** leads to the formation of large remnants
  - Particle-based N-body gravity calculations using the Barnes-Hut Tree Algorithm

# Physical model: Fracture and Pressure

The fracture and pressure model described below follows the approaches of **Benz (1995)** and **Benz & Asphaug (1999)**.

Tillotson EOS for the equation of state.

$$P = \left( a + \frac{b}{\frac{E}{E_0 \eta^2} + 1} \right) \rho E + A \mu + B \mu^2, \quad \eta = \frac{1}{\det(F)}, \quad \mu = \eta - 1.$$

- which is popular for SPH's numerical simulations of impacts.
- The volume change term is calculated using the deformation gradient tensor  $F$  rather than recalculating it.

The number density of flaws is given by the Weibull distribution (**Weibull 1939**).

- According to Weibull distribution, the number of flaws density is given by:  $n(\varepsilon) = k\varepsilon^m$ , where  $\varepsilon$  is local strain
- Each particle has multiple thresholds that can activate cracking.
- They are randomly distributed among the particles.

$$P_m = \begin{cases} (1 - \mathbf{D})P & \text{if } P < 0, \\ P & \text{if } P > 0 \end{cases}$$

The damage parameter  $D$  ranges from 0 to 1.

- When the local strain exceeds one or more strain thresholds of flaws, the associated cracks are activated and begin to grow.
- This parameter evolves continuously, propagating at half the speed of sound until  $D_{\max}$  is reached.
- $D_{\max}$  is a variable representing the fraction of flaw strain thresholds that are exceeded by the local strain, and it is updated every frame.
- $D$  weakens the material's response to expansion by adjusting the pressure according to the formula shown on the right.

# Physical model: Elastic deformation

For elastic response, we use the shear stress term from **the Neo-Hookean model**, instead of the deviatoric stress tensor commonly used in SPH. And we also exclude the pressure-related component of the stress calculated from it.

- Describes how intact material resists shape deformation through the deformation gradient  $\mathbf{F}$ .
- The logarithmic-strain-based constitutive model.
- The Neo-Hookean energy density model, adopted from **Mast et al. (2013)**, is given by:

$$\Psi(\mathbf{F}) = \mu \operatorname{tr}(\ln \boldsymbol{\Sigma}) + \frac{1}{2} \lambda (\operatorname{tr}(\ln \boldsymbol{\Sigma}))^2, \quad \mathbf{F} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^T$$

- The blue term in the constitutive model is the shear term.
- The Neo-Hookean model is widely used in MPM numerical simulations.
- In Drucker-Prager implementation via deformation gradient  $\mathbf{F}$ , deviatoric stress is replaced by Neo-Hookean shear stress.

***In our formulation, volume change effects are handled by the Tillotson EOS, so the Neo-Hookean model is used only for shear response.***

# Physical model: Drucker-prager elastoplasticity

For damaged particles, we apply Drucker–Prager elastoplasticity, following **Klár et al. (2016)**.

- Drucker-Prager plasticity applies to full damaged ( $D=1$ ) rock particles.
- This model adds **sand-like plasticity** to elastic bodies by modifying stresses.
- The Drucker-Prager model is applied to the **deformation gradient tensor  $\mathbf{F}$**  in our MPM code.
- This model uses a **non-associative flow rule**, which alters the particles' deformation gradient  $\mathbf{F}$  without changing its determinant—preserving volume during plastic flow.
- For partially damaged rock ( $0 < D < 1$ ), we use a pressure dependent yield strength according to **Jutzi (2015)**.

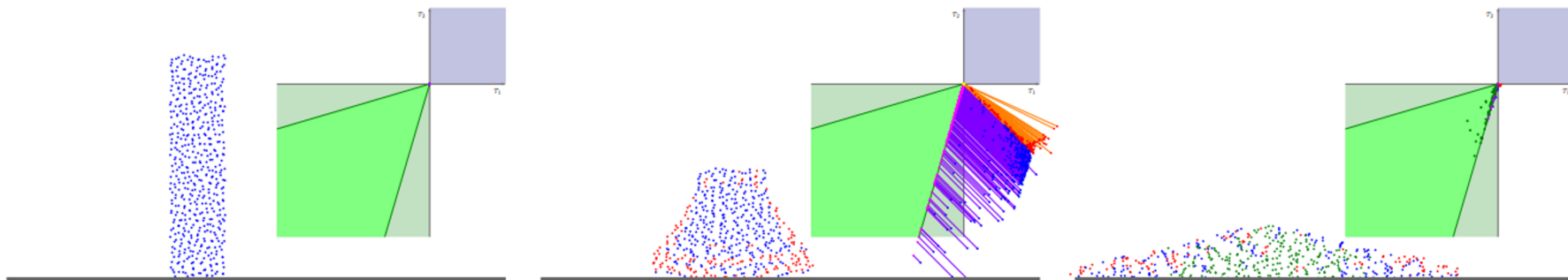
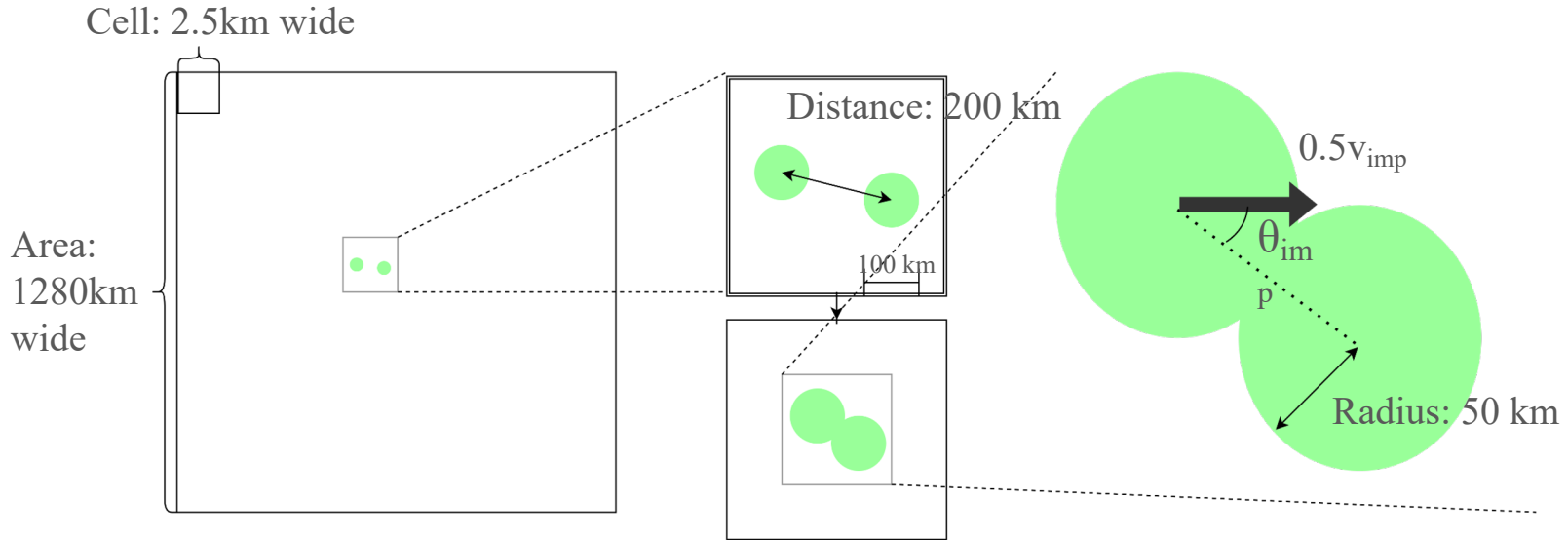


Figure from Klár et al., 2016, ACM Transactions on Graphics

# Simulation setup



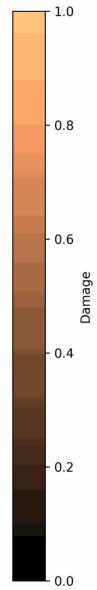
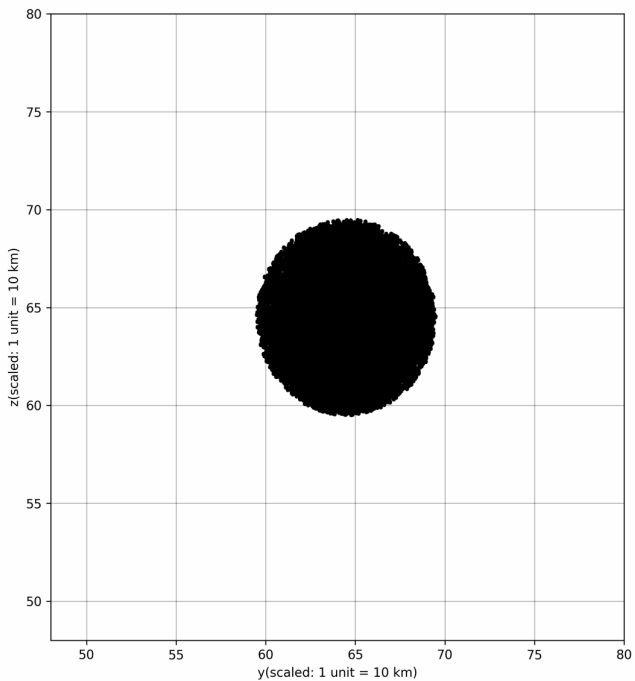
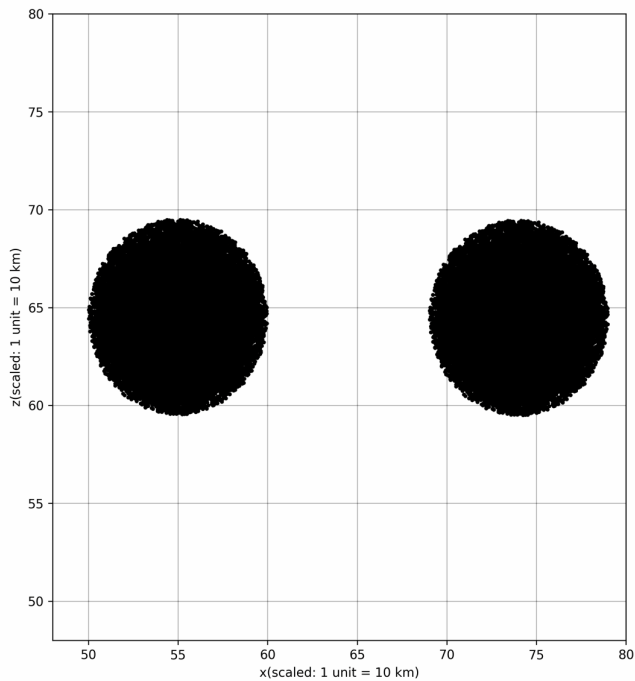
- $v_{imp}$  is impact velocity,  $\theta_{imp}$  is impact angle.
- Experimental conditions are based on **Sugiura et al. (2018)**.
- Number of particles is 20,000 per asteroid; 40,000 in total.
- Number of meshes is  $512^3$ .
- All simulation are operated on NVidia GTX Series GPU.

# Demonstration

**Simulation time = 15200s,  $M_{lr}/M_{target} = 1.743$ ,  $b/a = 0.89$ ,  $c/a = 0.85$ , impact velocity = 200m/s, angle = 5°**

*( $M_{lr}$ : the mass of largest remnants,  $M_{target}$ : the mass of an initial asteroid,  $a$ : major axis length,  $b$ : intermediate axis length,  $c$ : minor axis length)*

Time: 0.00 [s]

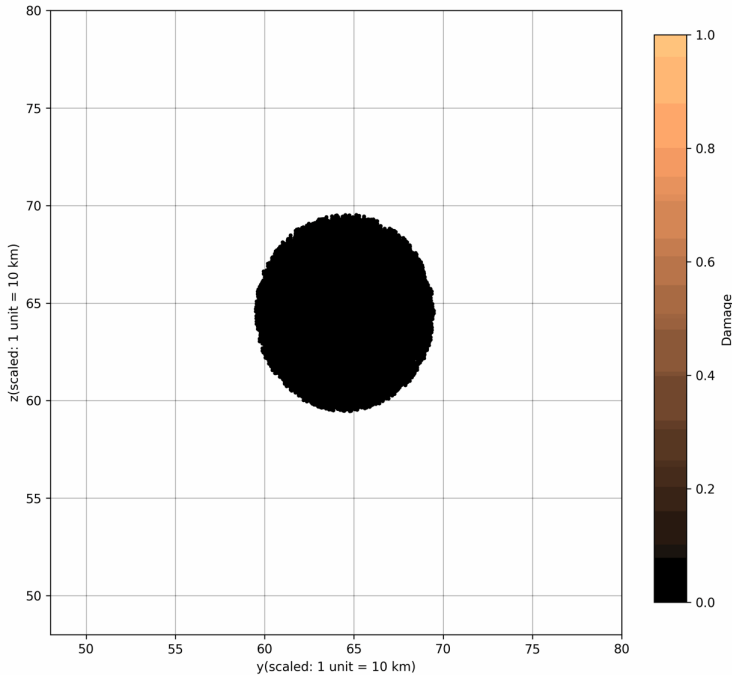
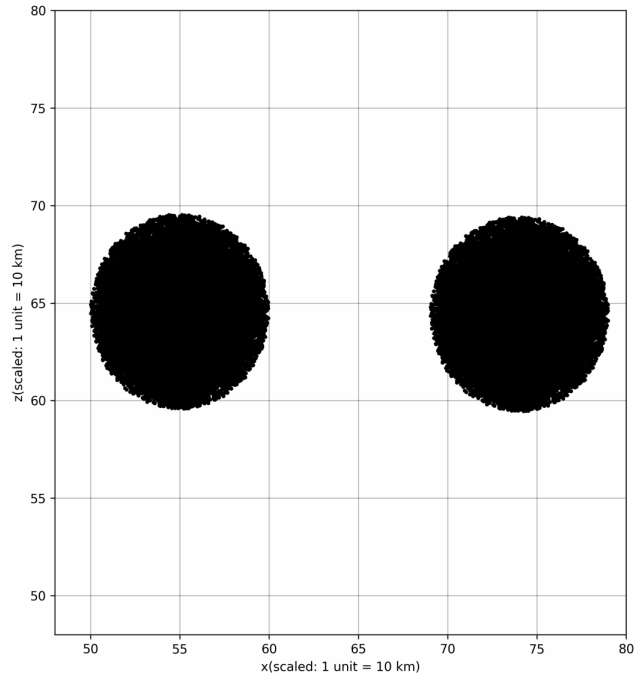


# Demonstration

**Simulation time = 10300s,  $M_{lr}/M_{target} = 1.737$ ,  $b/a = 0.32$ ,  $c/a = 0.27$ , impact velocity = 200m/s, angle = 15°**

*( $M_{lr}$ : the mass of largest remnants,  $M_{target}$ : the mass of an initial asteroid,  $a$ : major axis length,  $b$ : intermediate axis length,  $c$ : minor axis length)*

Time: 0.00 [s]

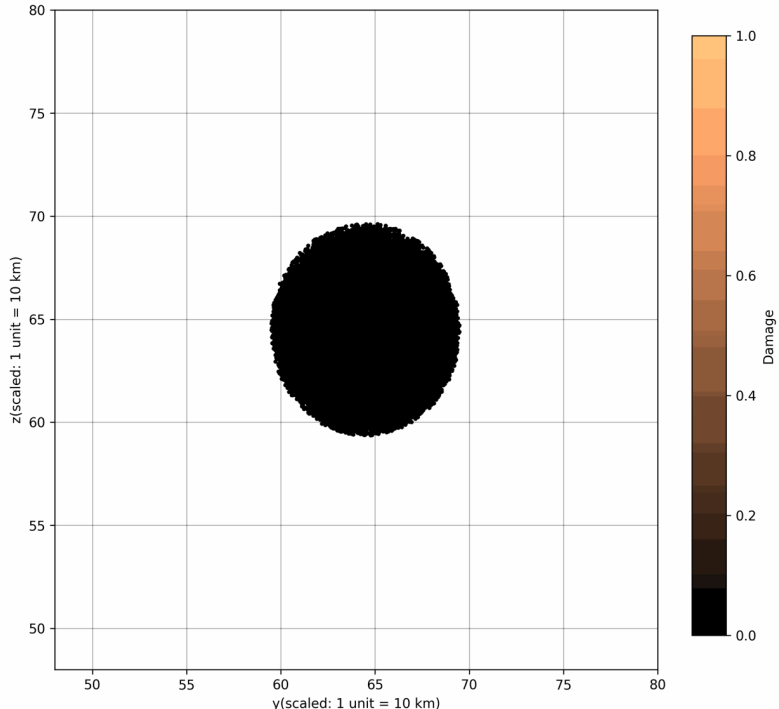
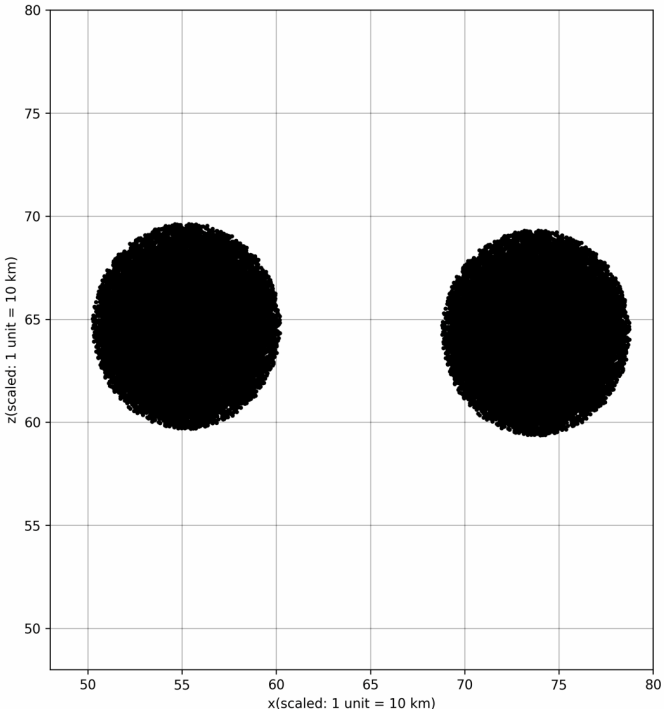


# Demonstration

**Simulation time = 5500s, impact velocity = 300m/s, angle = 25°**

*(M<sub>lr</sub>: the mass of largest remnants, M<sub>target</sub>: the mass of an initial asteroid, a: major axis length, b: intermediate axis length, c: minor axis length)*

Time: 0.00 [s]



# Summary and Future work

- We ran a simulation under the same experimental setup as a previous asteroid impact study, using both MPM and the Barnes-Hut algorithm.
- Both methods were optimized for GPU, enabling realistic simulation times.
- We modified a constitutive model originally designed for SPH, making it compatible as a general variable in MPM.
- We introduce several demonstration cases with different impact velocities and angles to illustrate how they affect the shape of the largest resulting fragment.
- We plan to extend our simulations to more experimental settings and present detailed results in future work.

# Physical model: Paramater

$k$ ( $m^{-3}$ )	<i>Weibull distribution</i>	$4.0 \cdot 10^{35}$
$m$	<i>Weibull distribution</i>	9.0

$\rho_0$ ( $kg/m^3$ )	<i>density of basalt</i>	2700
$A$ ( $J/m^3$ )	<i>Tillotson EOS</i>	$2.67 \cdot 10^{10}$
$B$ ( $J/m^3$ )	<i>Tillotson EOS</i>	$2.67 \cdot 10^{10}$
$a$	<i>Tillotson EOS</i>	0.5
$b$	<i>Tillotson EOS</i>	1.5
$E_0$ ( $J/kg$ )	<i>Tillotson EOS</i>	$4.87 \cdot 10^8$

$\mu$ ( $J/m^3$ )	<i>the shear module</i>	$2.27 \cdot 10^{10}$
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friction coefficient ( $^\circ$ )	40
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# Material Point Method (MPM): Discrete Eqs (Appendix)

interpolation weights is quadratic splines:

$$N(x) = \begin{cases} \frac{3}{4} - x^2 & \text{if } 0 \leq |x| < \frac{1}{2}, \\ \frac{(3/2 - |x|)^2}{2} & \text{if } \frac{1}{2} \leq |x| < \frac{3}{2}, \\ 0 & \text{if } \frac{3}{2} \leq |x|. \end{cases}$$

$$\omega_{ip} = N(|x_p - x_i|) N(|y_p - y_i|) N(|z_p - z_i|)$$

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particle to grid (P2G):

$$m_i^n = \sum_p \omega_{ip}^n m_p$$

$$\mathbf{mv}_i^n = \sum_p \omega_{ip}^n m_p (\mathbf{v}_p^n + \nabla \mathbf{v}_p^n \cdot (\mathbf{I} + \Delta t \nabla \mathbf{v}_p^n)^{-1} \cdot (\mathbf{x}_i^n - \mathbf{x}_p^n))$$

$$\mathbf{f}_i^n(\mathbf{F}^n) = - \sum_p V_p \sigma(\mathbf{F}^n) \nabla \omega_{ip}^n$$

Update grid:

$$\mathbf{mv}_i^{n+1} = \mathbf{mv}_i^n + \Delta t \mathbf{f}_i^n$$

$$\mathbf{v}_i^{n+1} = \frac{\mathbf{mv}_i^{n+1}}{m_i^n}$$

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grid to particle (G2P):

$$\mathbf{v}_p^{n+1} = \sum_p \omega_{ip}^n m_p (\mathbf{v}_i^{n+1})$$

$$\nabla \mathbf{v}_p^{n+1} = \left( \sum_i \omega_{ip} \mathbf{v}_i (\mathbf{x}_i - \mathbf{x}_p)^\top \right) \cdot \left( \sum_i \omega_{ip} (\mathbf{x}_i - \mathbf{x}_p) (\mathbf{x}_i - \mathbf{x}_p)^\top \right)^{-1}$$

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Update deformation gradient  $\mathbf{F}$ :

$$\mathbf{F}_p^{n+1} = (\mathbf{I} + \Delta t \nabla \mathbf{v}_p^{n+1}) \mathbf{F}_p^n$$

# Physical model: Brittle failure (Appendix)

The number density of flaws is given by the **Weibull distribution**.

- **Weibull distribution** is given by:
  - $n(\varepsilon) = k\varepsilon^m$ , where  $\varepsilon$  is local strain
- Each particle has multiple thresholds that can activate cracking.
- The total number of thresholds is estimated as:
  - $N_f \approx 2 \cdot N_{part} \cdot \ln(N_{part})$ , where  $N_{part}$  is the number of particles.
- They are **randomly** distributed among the particles.
- j-th threshold strain values are given by:
  - $\varepsilon_j = [j / (kV)]^{1/m}$ , where  $1 \leq j \leq N_f$

**The damage parameter D** ranges from 0 to 1.

- It increases when the local stress exceeds one or more thresholds.
- Maximum damage is defined as:
  - $D_{max} = N_a / N_{max}$ , where  $N_a$  is number of thresholds exceeded by particle strain and  $N_{max}$  is total number of thresholds assigned to the particle
- The evolution of damage follows:
  - $dD^{1/3} / dt = c_g / R$

# Physical model: Energy density (Appendix)

Energy density is used in Tillotson EOS.

- It is updated using the following relation:
  - $dE/dt = 1/\rho \cdot \boldsymbol{\sigma} : \mathbf{d}$
  - where  $\mathbf{d}$  is the rate of deformation tensor, given by:  $\mathbf{d} = 0.5(\nabla \mathbf{v} \cdot (\nabla \mathbf{v})^T)$