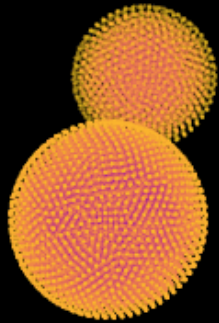


# Primordial Planetesimal Collisions with Dust Aggregate Equation of State



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**Collaborator:  
Hiroshi Kobayashi**

# Background – Collisions in Planet Formation

2

## Key Points:

Collisions is important for understanding planet formation

## Traditional approach:

*Impact energy vs. Binding energy*  
(*impact velocity*)                      (*cohesion + gravitation*)

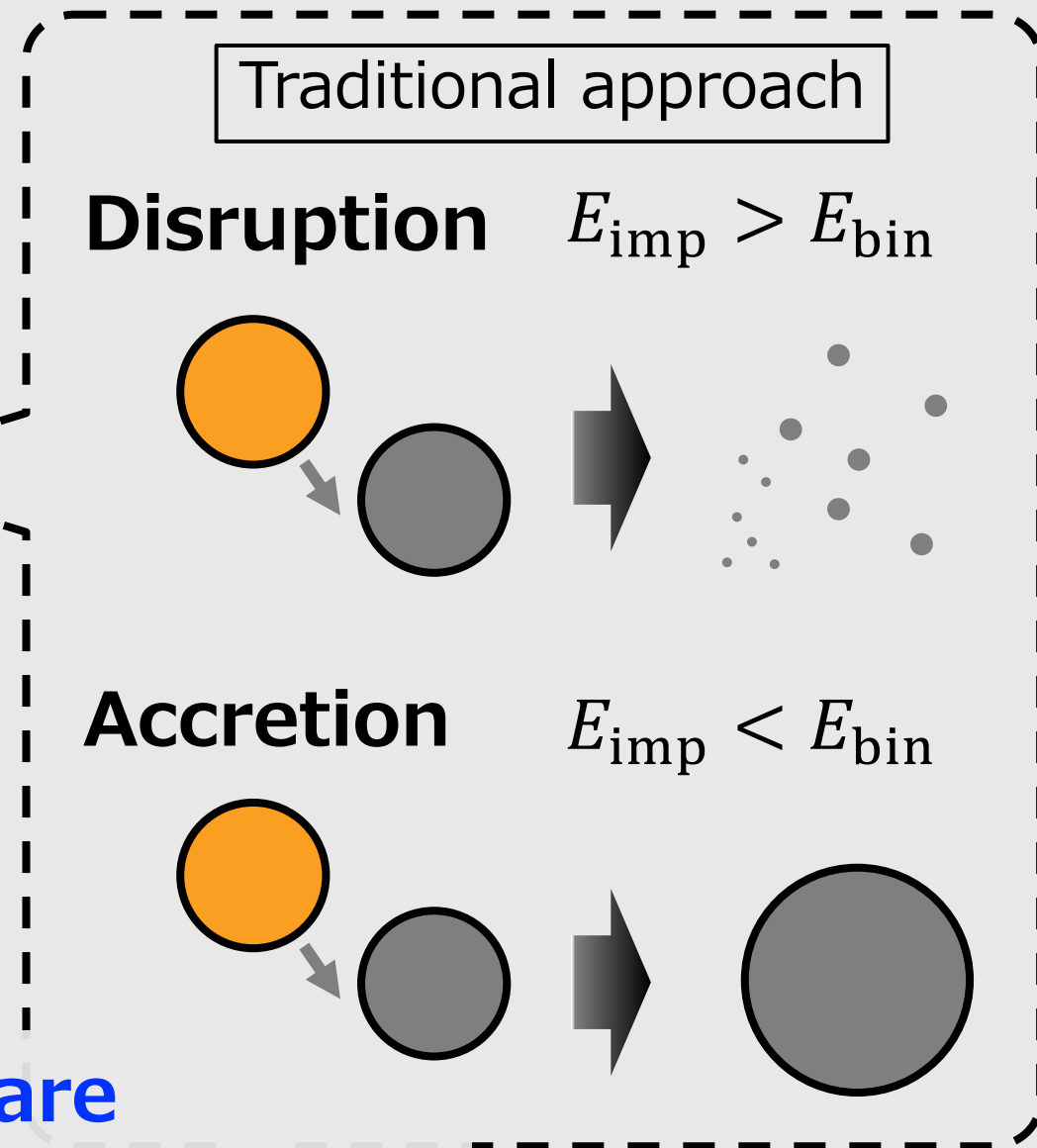
➔ **Oversimplified**

## Limitations:

Ignores the dissipation process  
(ex. friction, partial destruction)

✗ Cannot address impact angle or mass ratio

✓ **Therefore, numerical simulations are required to properly study realistic collisions.**



# Recent progress & their problems

3

## Recent progress

- Elastic-body SPH + fiction model (Sugiura18, Jutzi+15)

→ Assumed **monolithic rock**

## Primordial planetesimals:

- Formed from **sub- $\mu\text{m}$  dust, no crystal structure**
- **Cohesive strength** due to dust grain interactions
- Properties differ from fluid or rock

## Limitations of previous studies:

**Neglected microscopic behavior of dust**

**This study:**

Understand collisional physics of primordial planetesimals Focus on **dust-aggregate planetesimals**

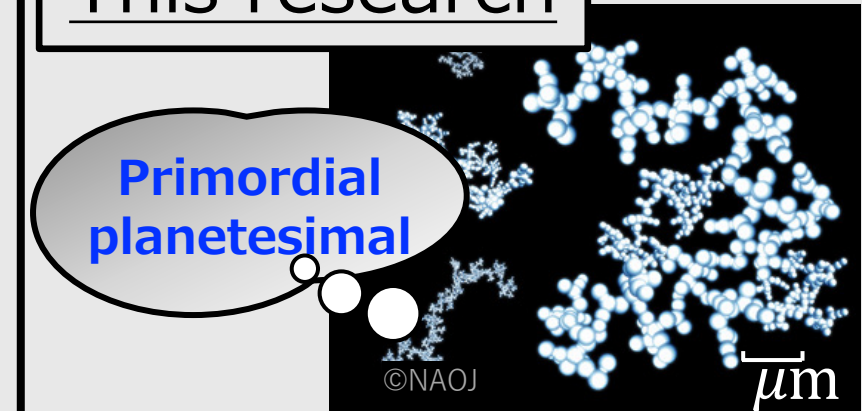
Previous research



**Monolithic rock**

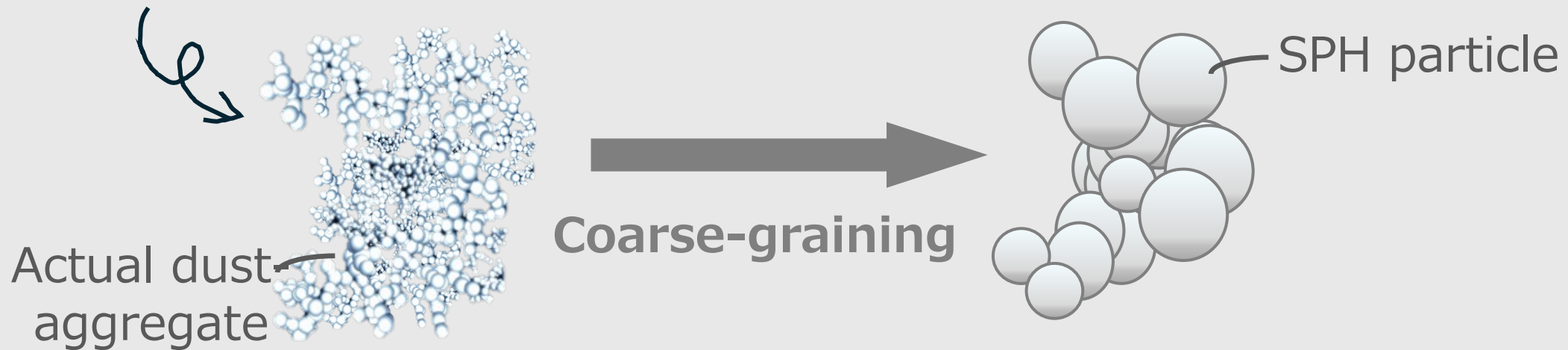
VS

This research



**Dust-aggregate**

- We assume SPH particles represent dust-aggregates



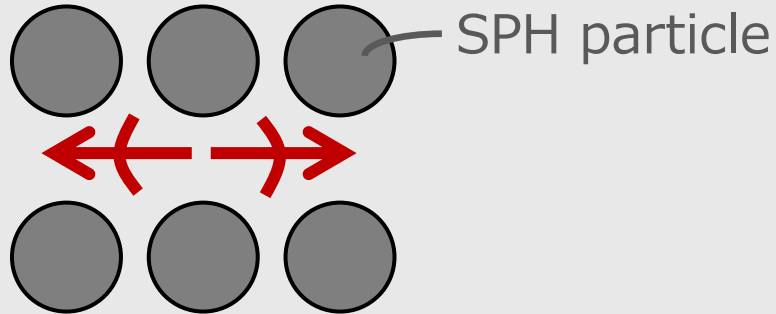
- Basic equations

$$\begin{cases} \frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v} \\ \frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P_{\text{total}} - \mathbf{g} \end{cases}$$

- In SPH formulation

$$\begin{cases} \rho_i = \sum_j m_j W(x_i - x_j, h_i) \\ \frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left( \frac{P_{\text{total},i}}{\rho_i^2} + \frac{P_{\text{total},j}}{\rho_j^2} + \Pi_{ij} \right) \nabla W(x_i - x_j, h_i) - \mathbf{g} \end{cases}$$

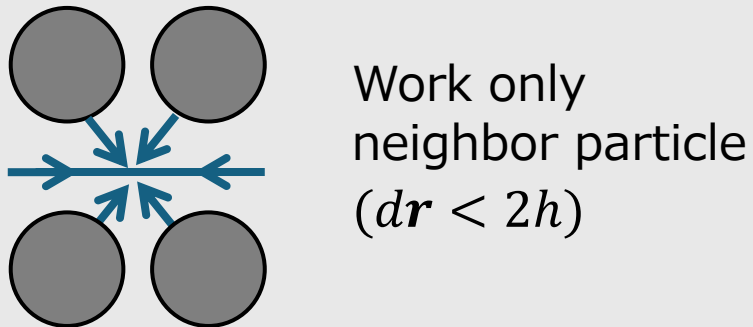
## Pressure (resist compaction)



$$P_{\text{comp}} = \frac{E_{\text{roll}}}{r_0^3} \left( \frac{\rho_0}{\rho} - \frac{\rho_0}{\rho_{\text{max}}} \right)^{-3}$$

(Tatsuuma+19)

## Negative pressure



$$P_{\text{tensile}} = C \frac{F_c}{r_0^2} \left( \frac{\rho}{\rho_0} \right)^{1.8}$$

(Tatsuuma+23)

## Our Pressure model

- When particles compressed:

$$P_{\text{total}} = P_{\text{comp}}$$

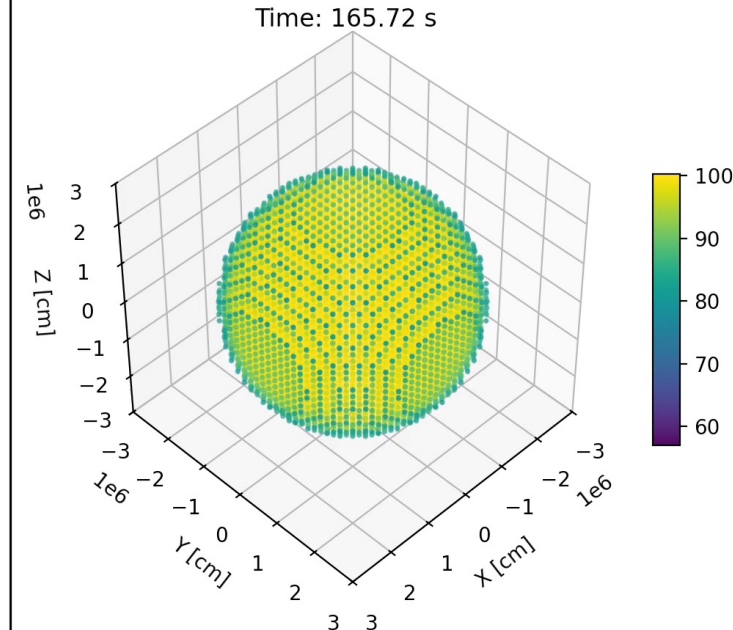
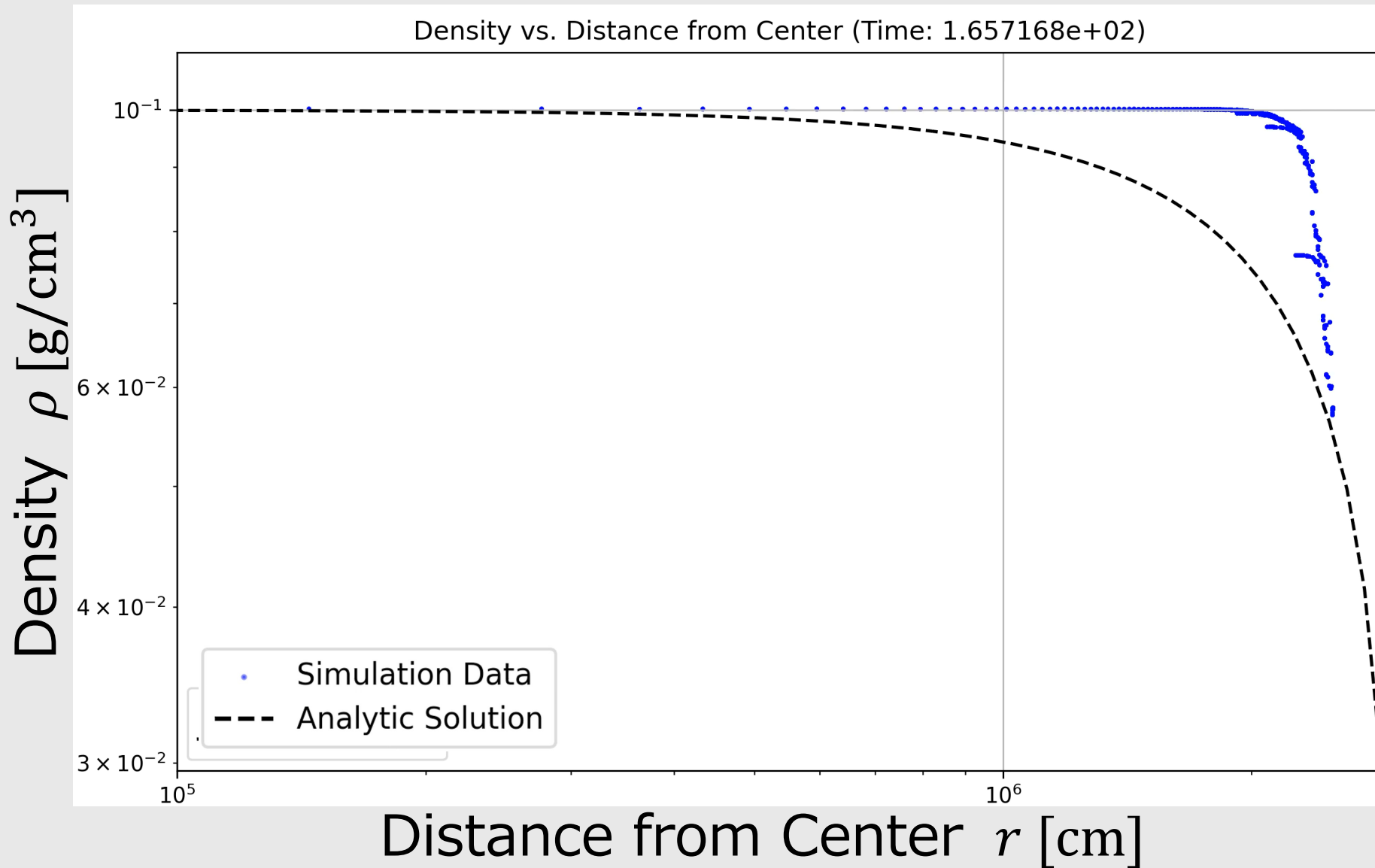
- When neighbor particles separated:

$$P_{\text{total}} = P_{\text{comp}} - P_{\text{tensile}}$$

- When the distance is far enough:

$$P_{\text{total}} = 0$$

$$\frac{dv_i}{dt} = - \sum_j m_j \left( \frac{P_{\text{total},i}}{\rho_i^2} + \frac{P_{\text{total},j}}{\rho_j^2} + \Pi_{ij} \right) \nabla W(x_i - x_j, h_i) - \mathbf{g}$$

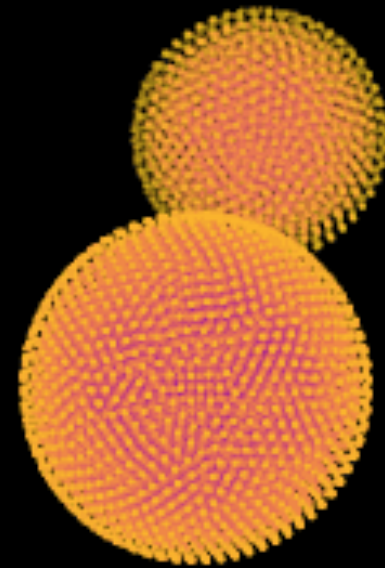


We verified the gravitational equilibrium solution for the dust-aggregate using the SPH method.

# Simulations of planetesimal collision

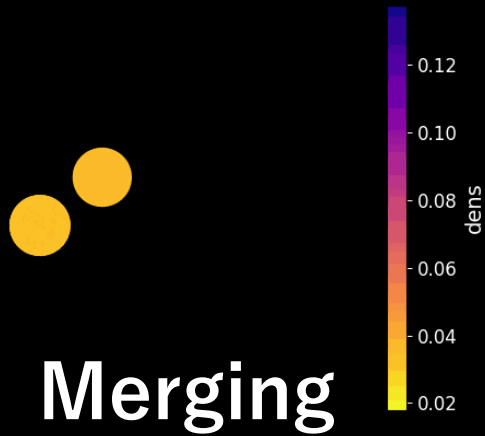
7

Target Radius : 25 km



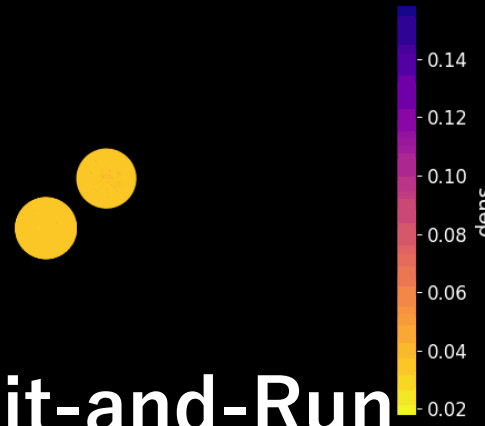
# Result – How to see graphs

Time = 0.00



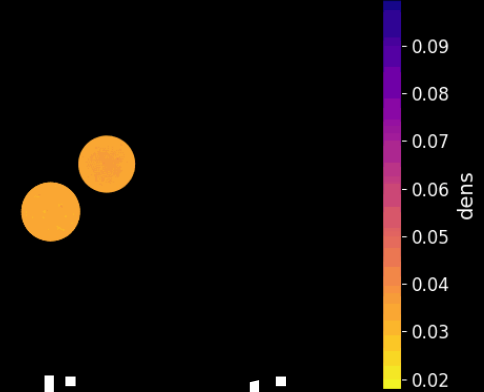
Merging

Time = 0.00



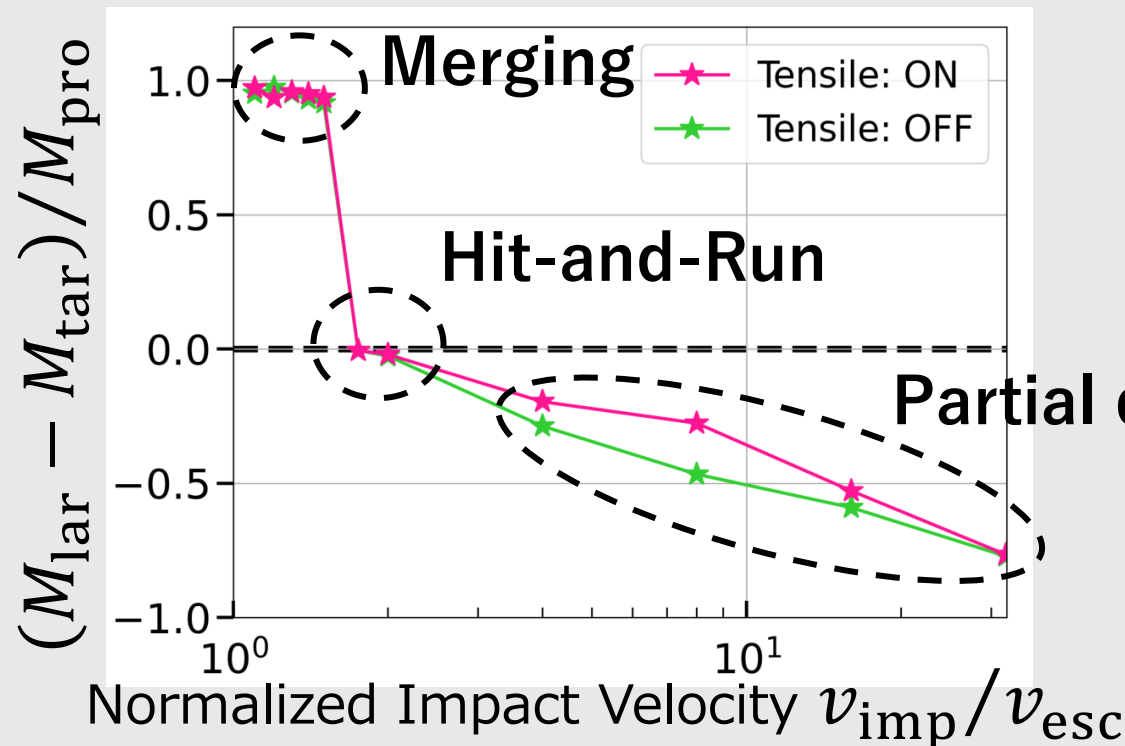
Hit-and-Run

Time = 0.00



Partial disruption

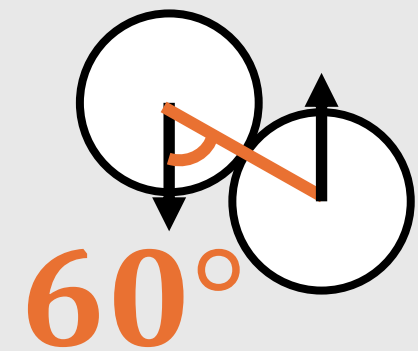
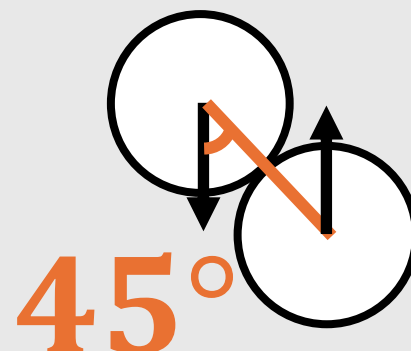
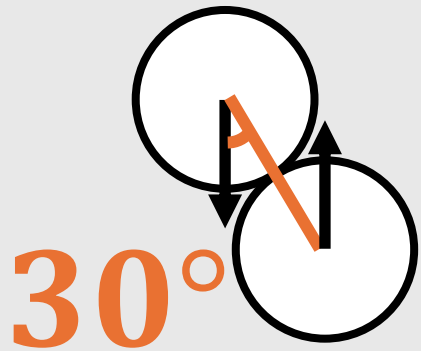
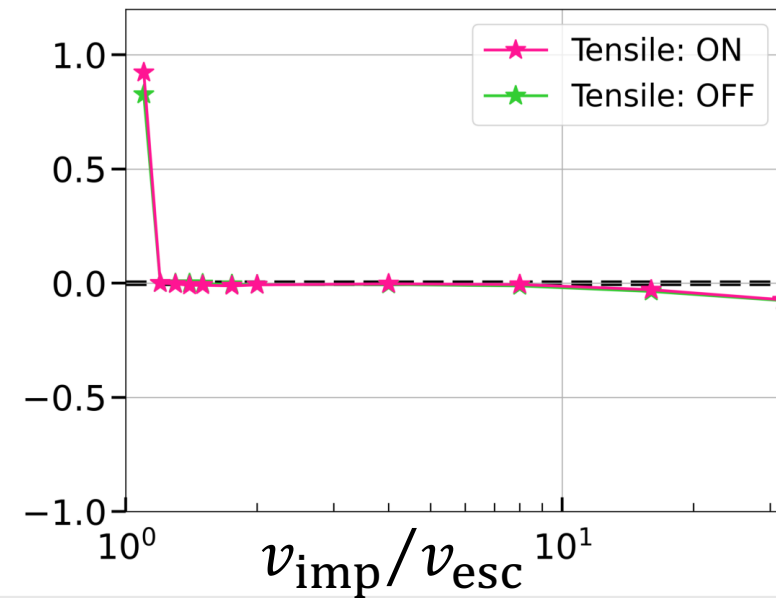
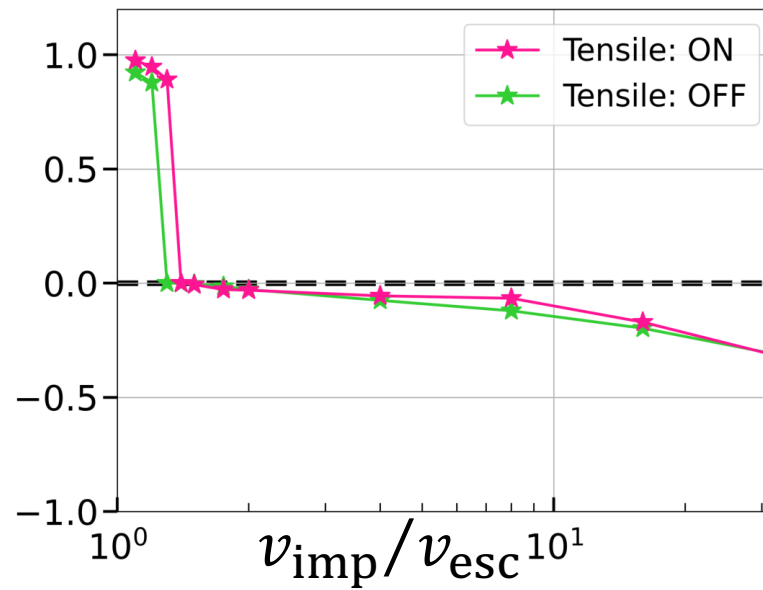
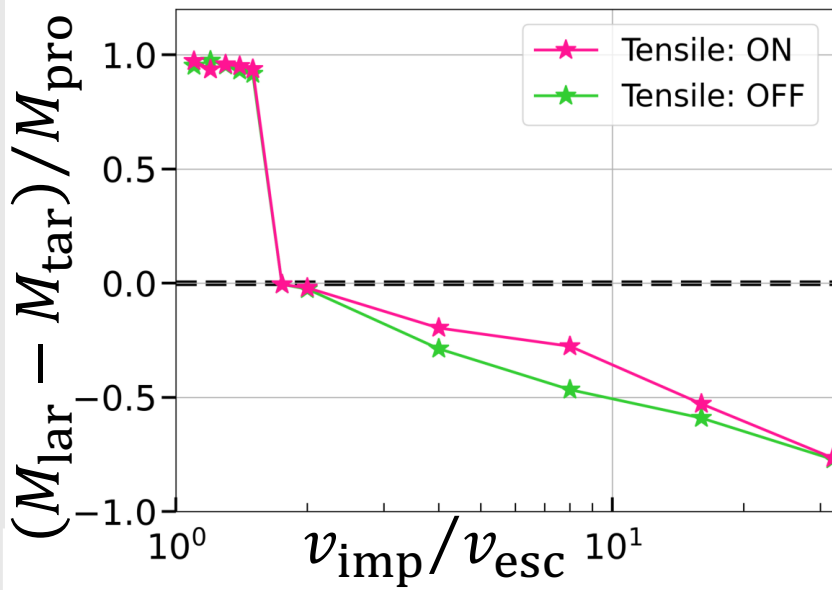
Roughly indicates the accretion fraction



# Result – Dependence on impact angle

Mass ratio 1:1

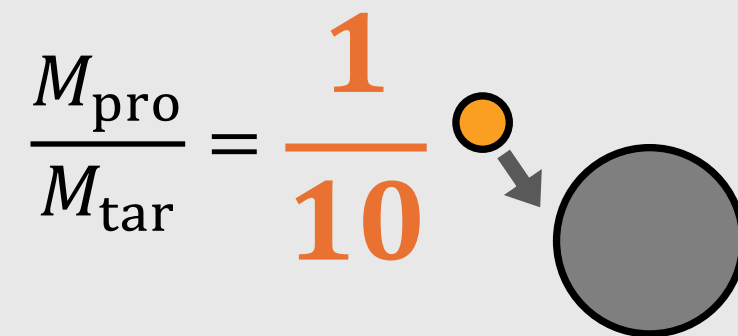
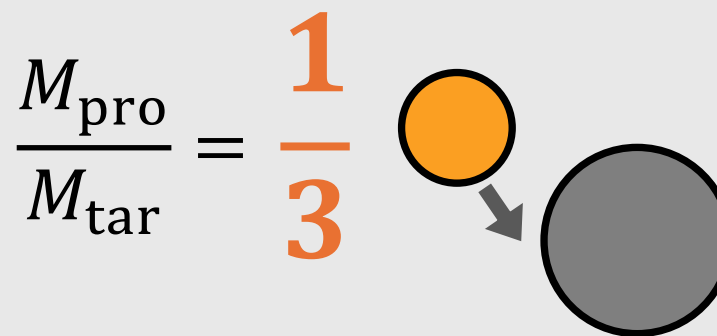
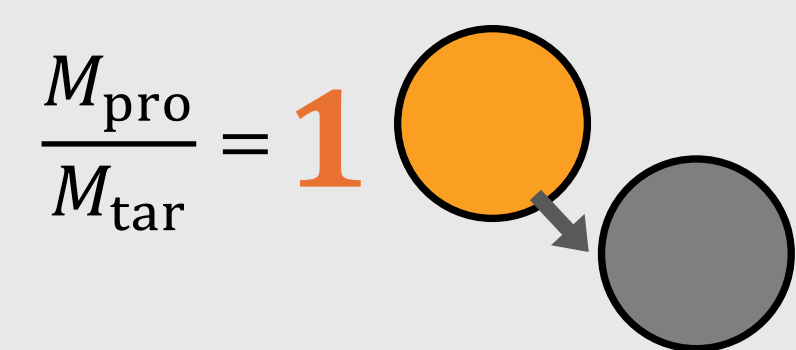
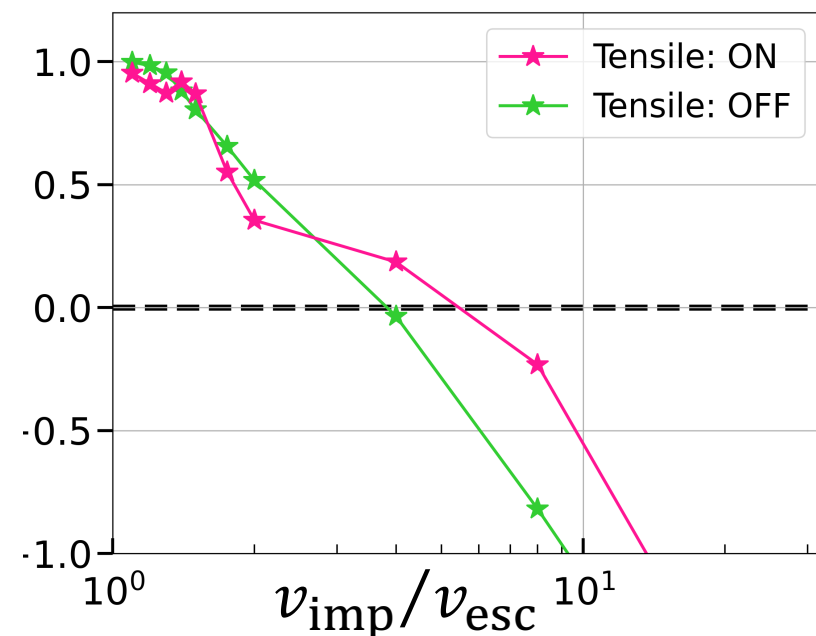
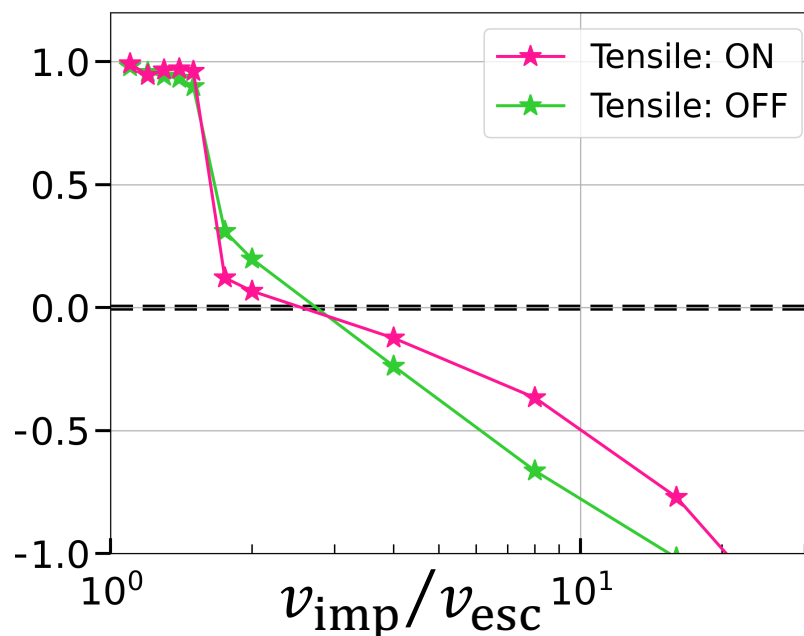
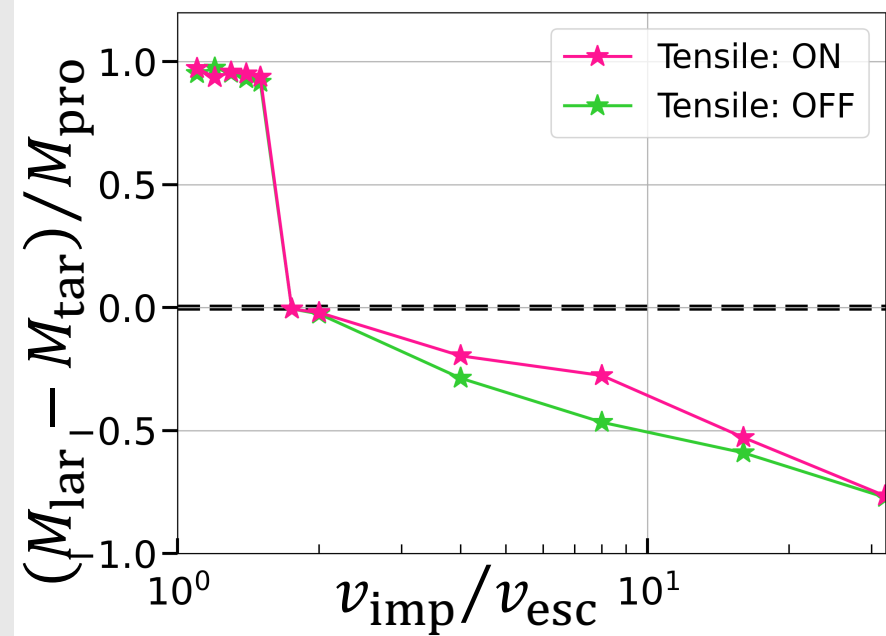
9



- 3 cases: 30°, 45°, 60°
- **More oblique** → **less disruptive** (survive at higher  $v/v_{\text{esc}}$ )
- **Cohesive dust-aggregate** model shows **milder disruption** than no-cohesion

# Result - Dependence on mass ratio

Impact angle: 30° **10**



- Disruption threshold **shifts to higher velocity** with Mass ratio
- With cohesion: less mass loss in high-velocity

## Future work

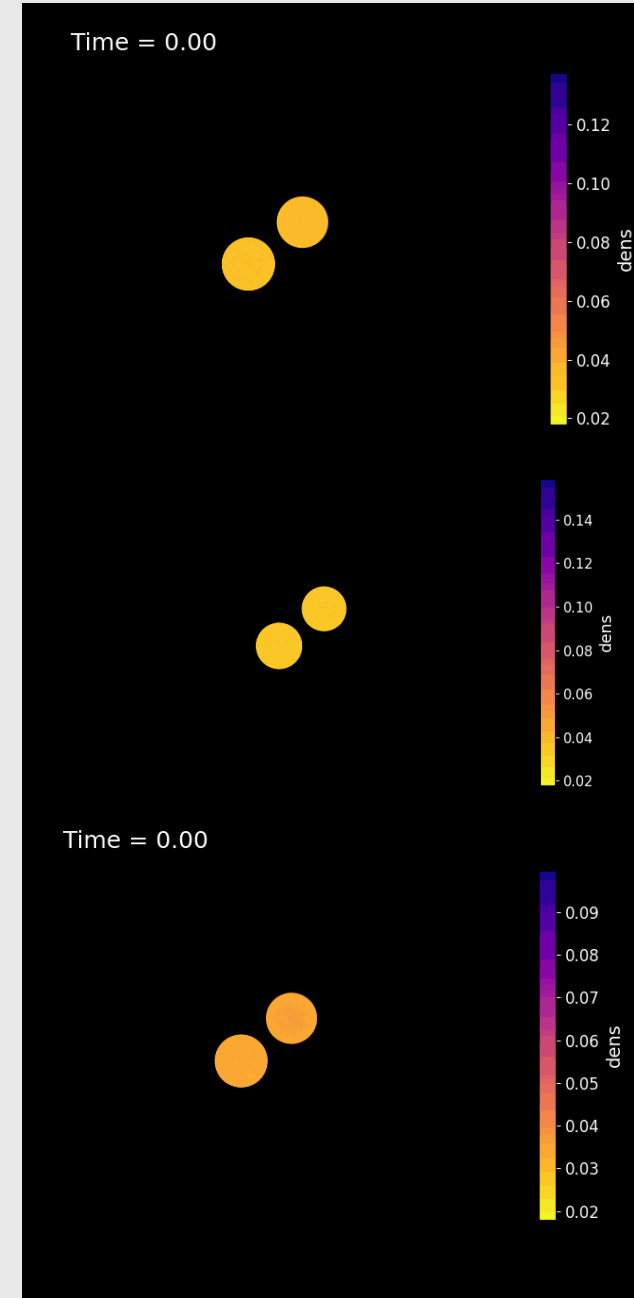
- Analytical understanding with dissipated energy (compression) and overlap geometry

# Summary

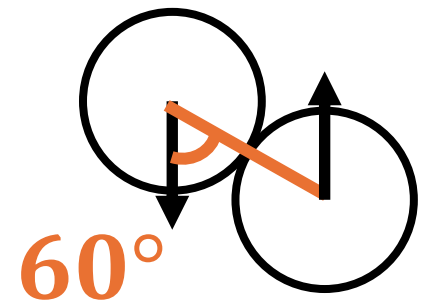
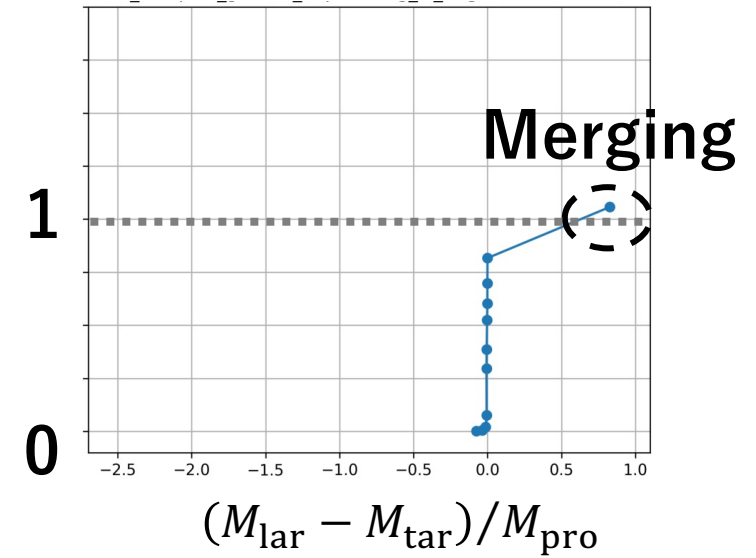
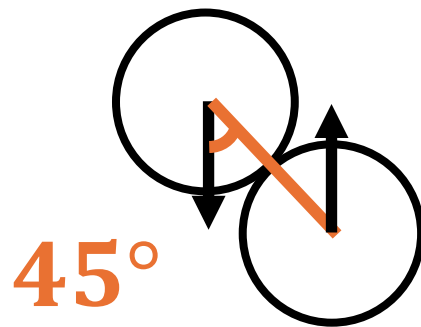
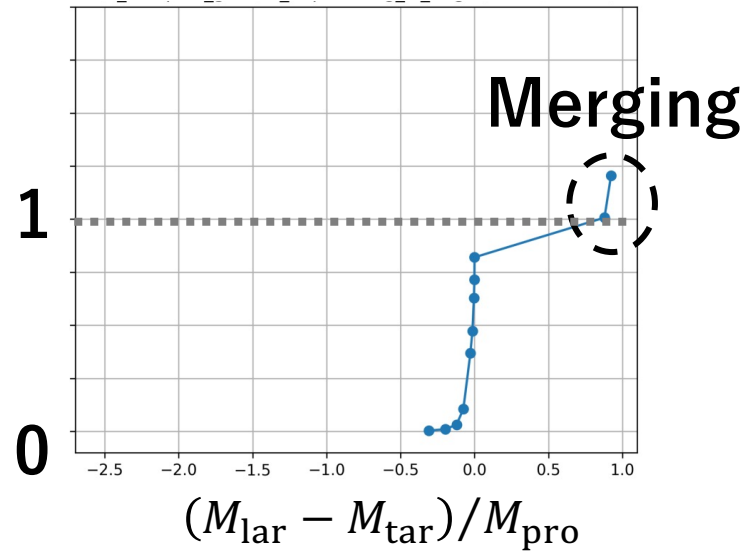
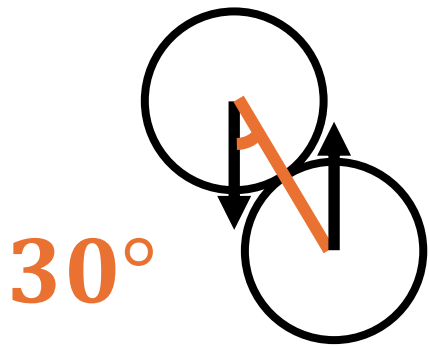
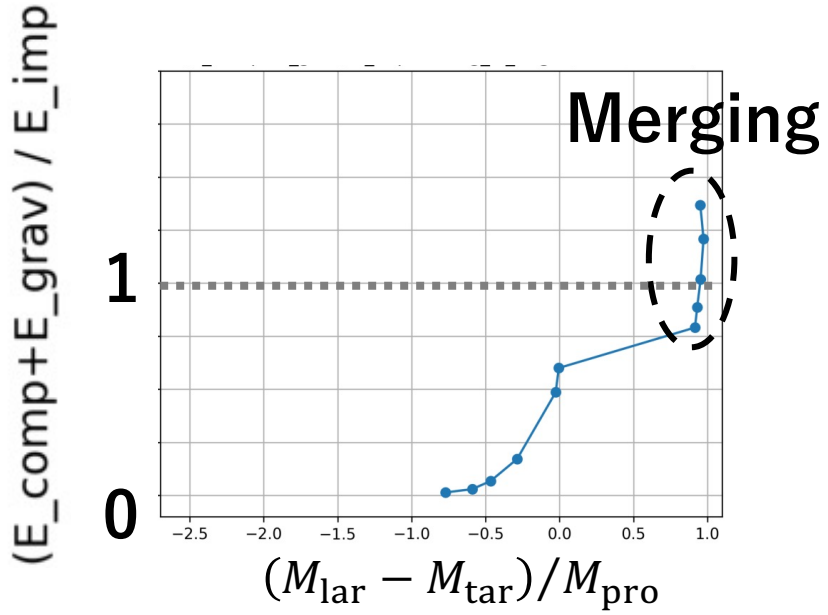
- Implemented a dust-aggregate equation of state and performed SPH simulations of planetesimal collisions.
- Correctly reproduced equilibrium shapes under self-gravity and pressure-gradient forces.
- Confirmed that dust cohesion strongly affects collision outcomes.

# Future work

- Develop a general collision model through energy dissipation and geometric analysis to understand the physics of primordial planetesimal collisions.

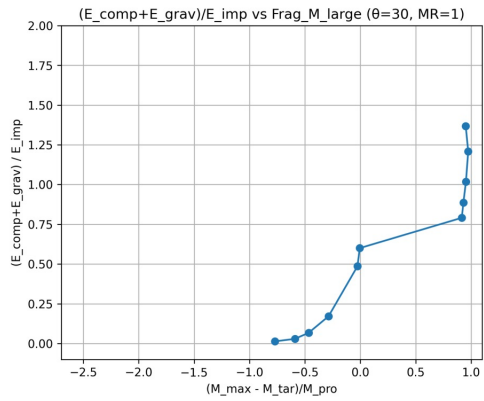


# Appendix

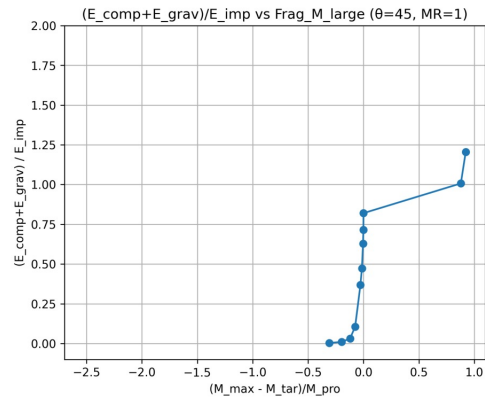




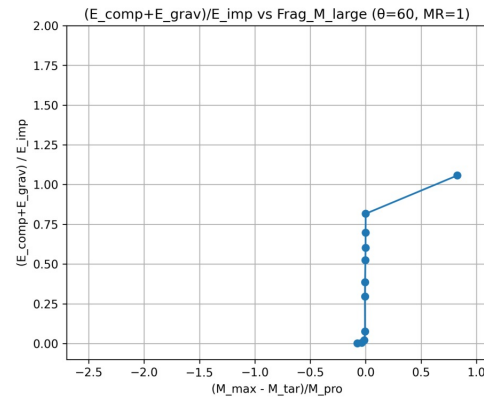
MR=1,  $\theta=30$



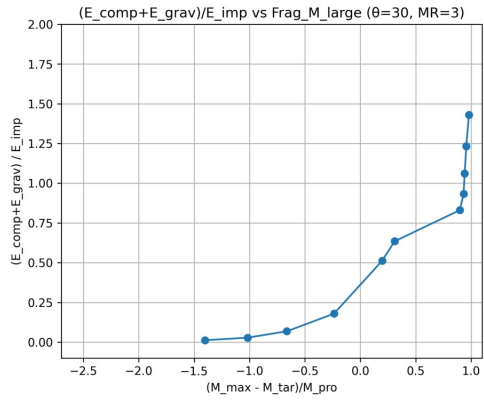
MR=1,  $\theta=45$



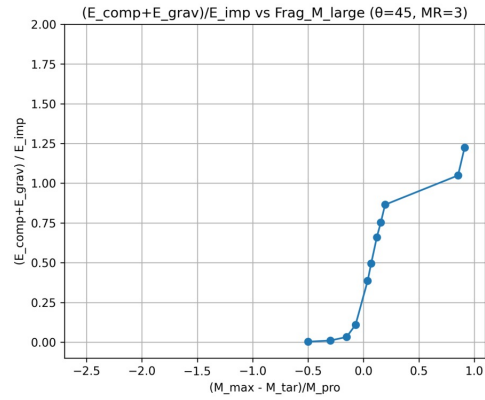
MR=1,  $\theta=60$



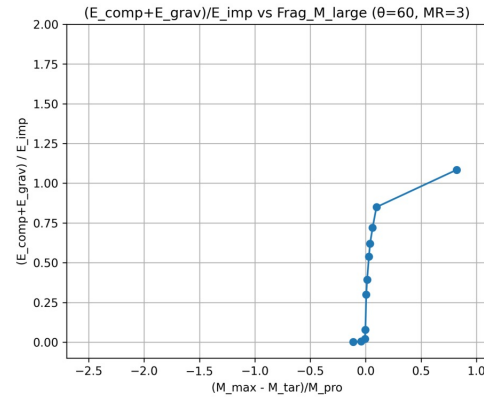
MR=3,  $\theta=30$



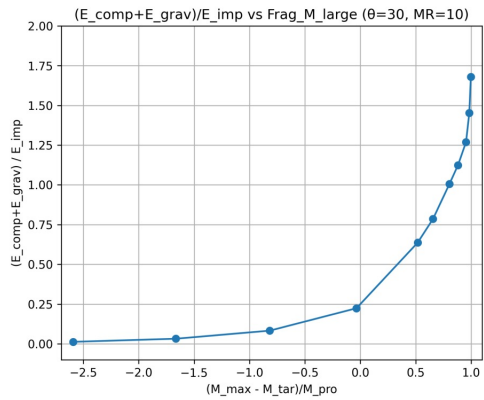
MR=3,  $\theta=45$



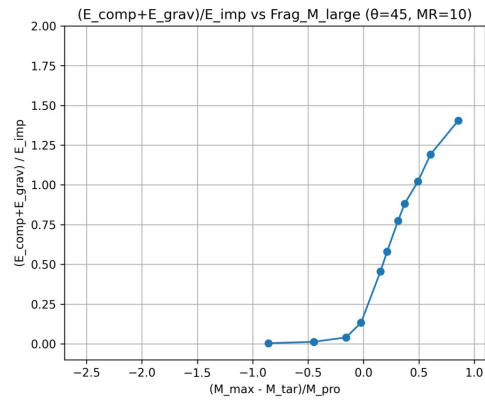
MR=3,  $\theta=60$



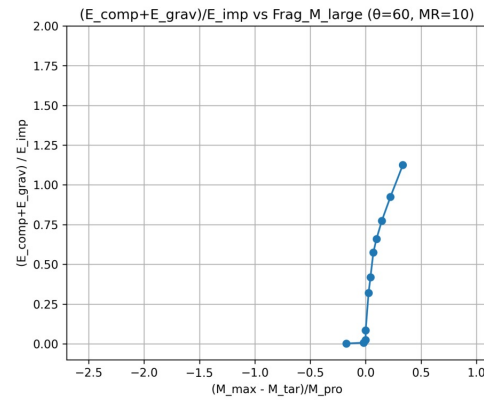
MR=10,  $\theta=30$



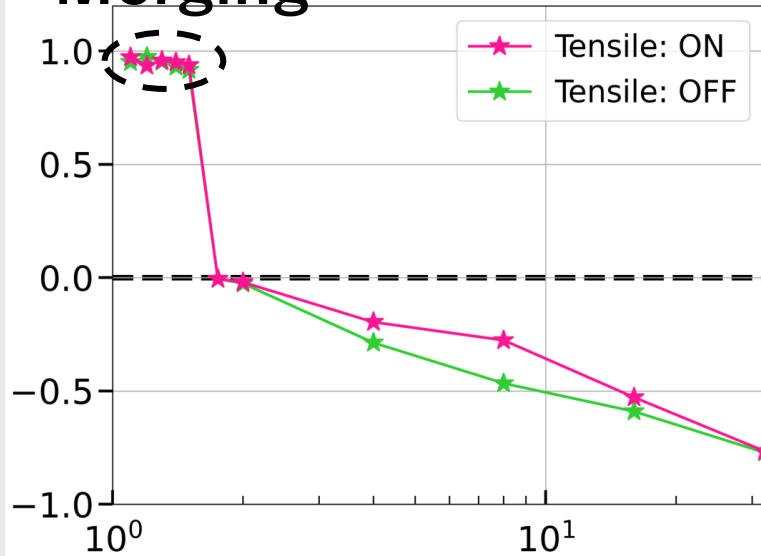
MR=10,  $\theta=45$



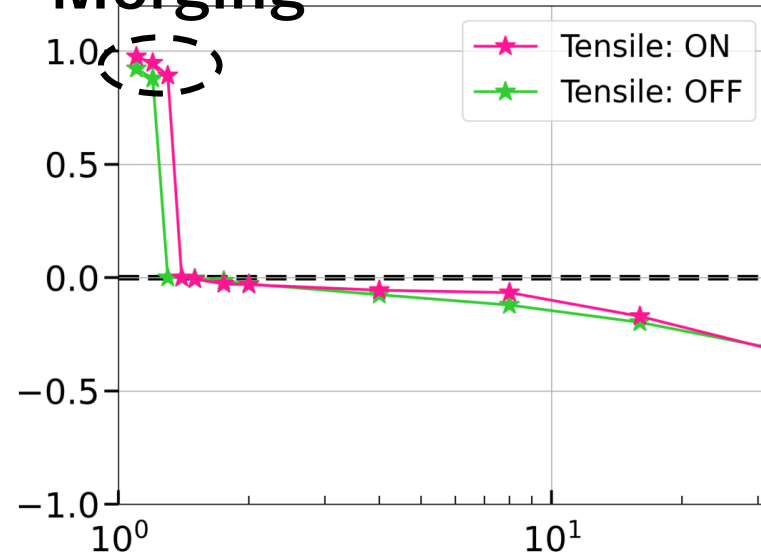
MR=10,  $\theta=60$



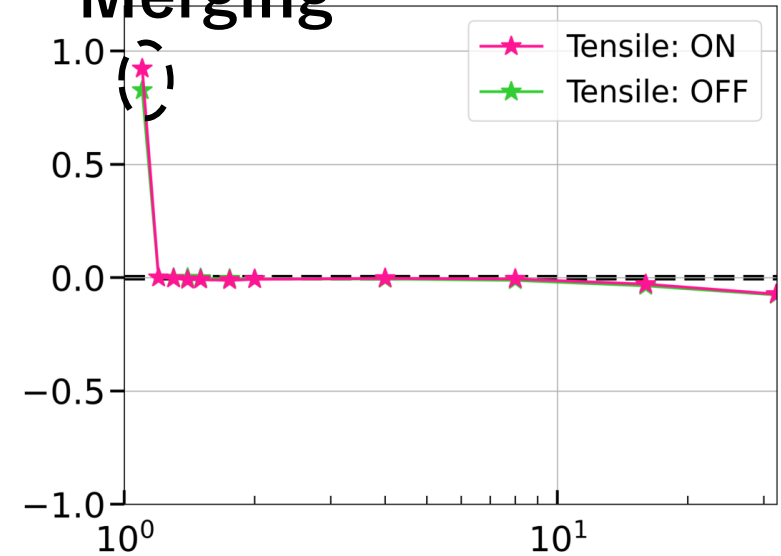
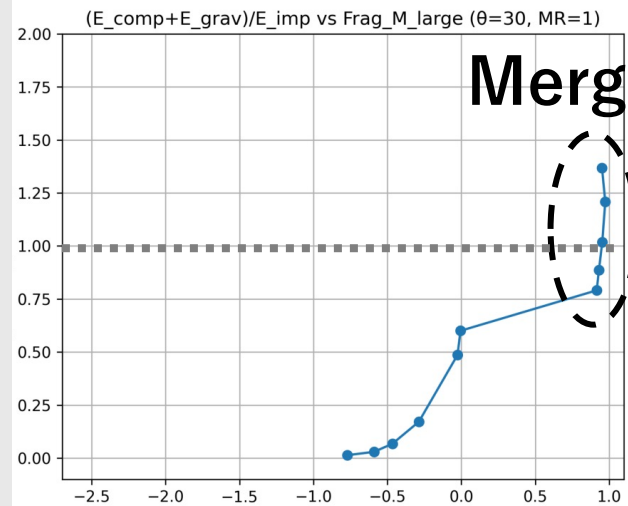
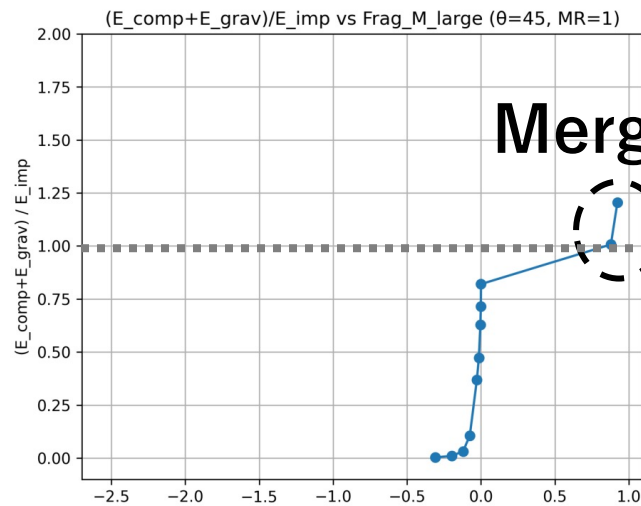
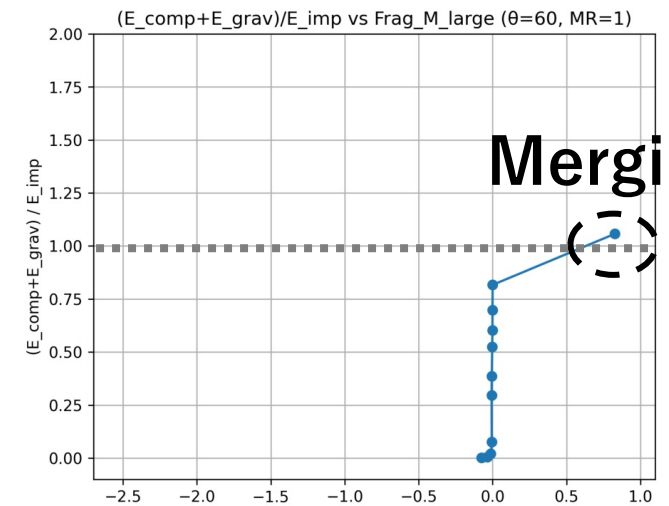
## Merging



## Merging

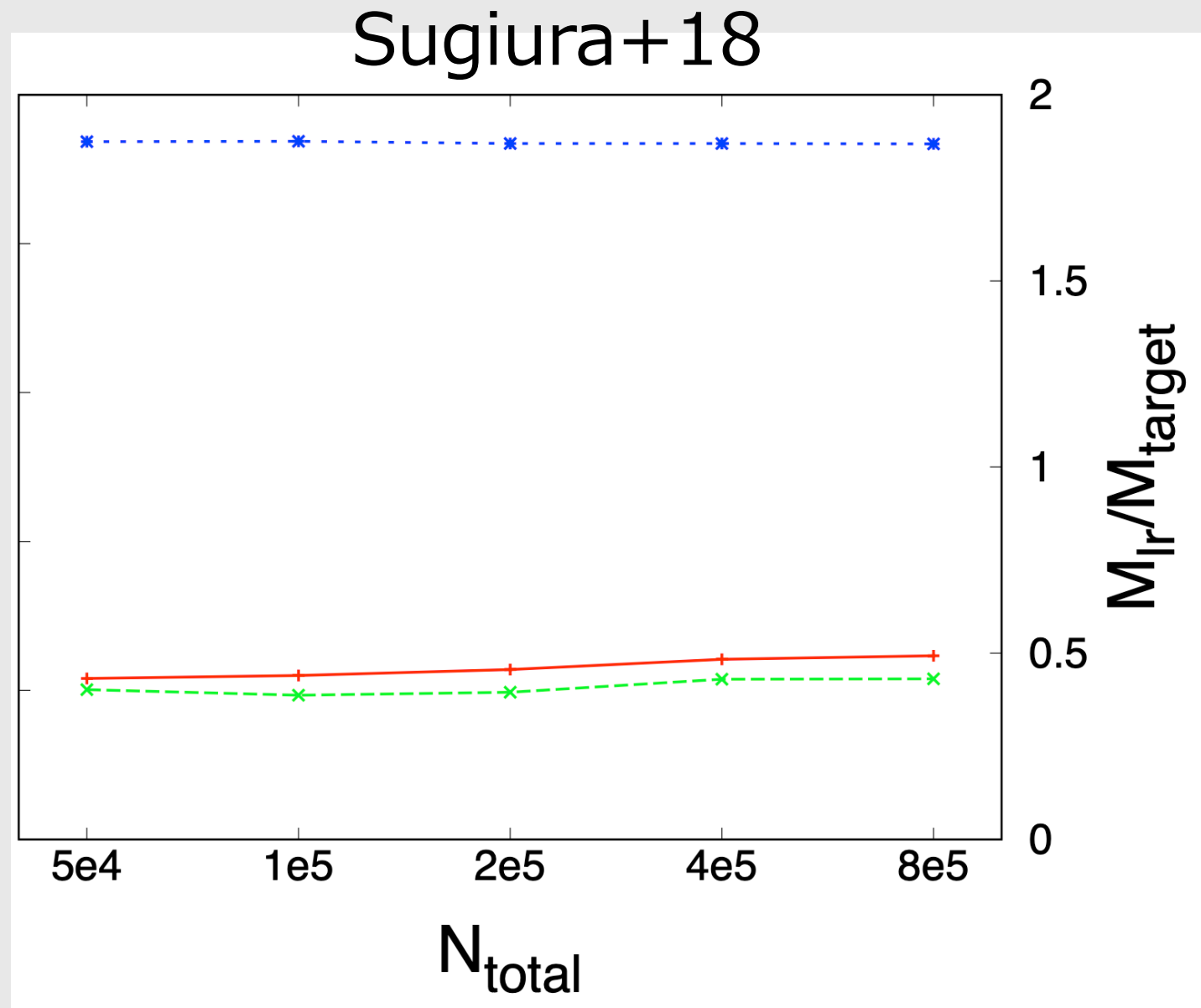


## Merging

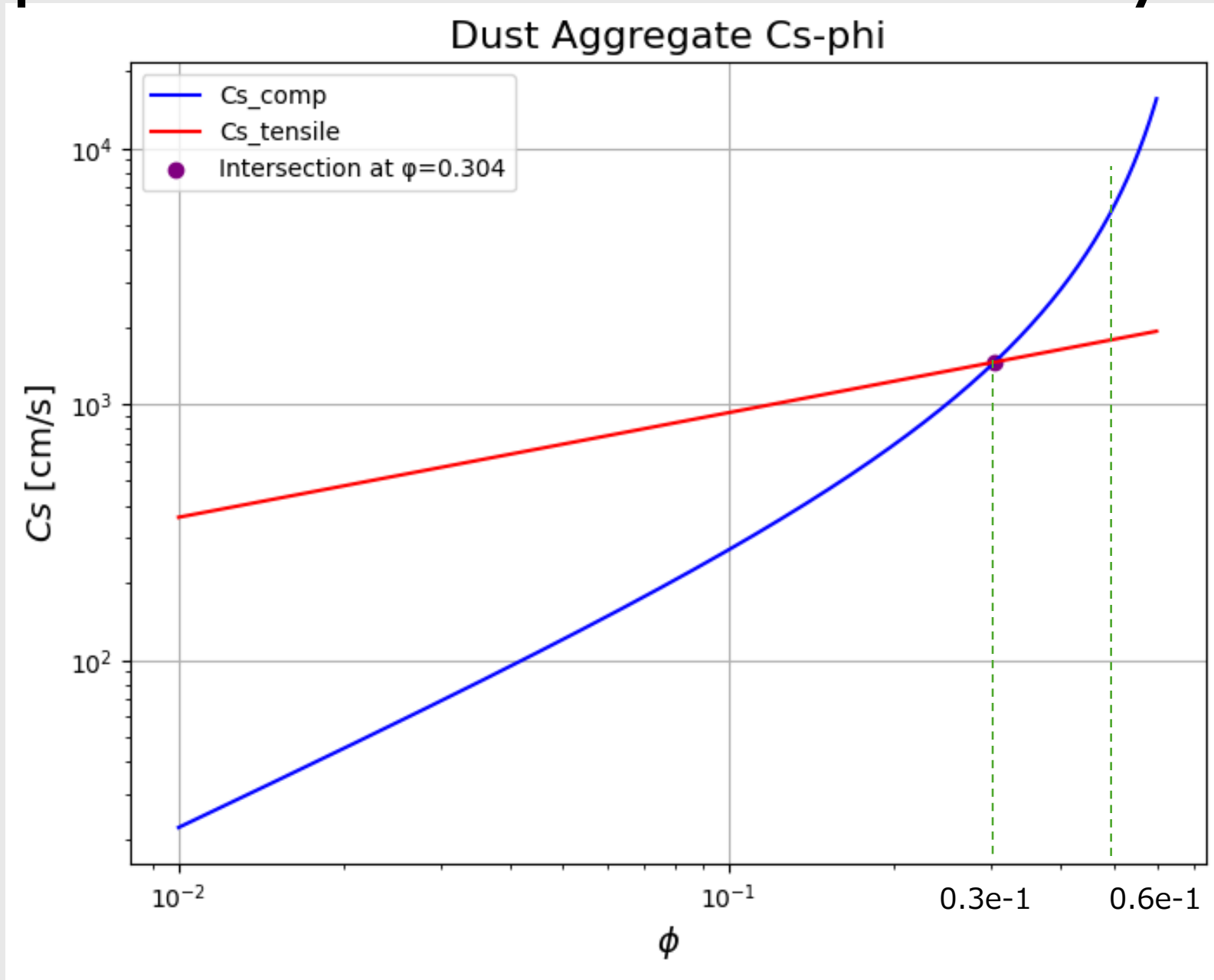
 $(E_{\text{comp}}+E_{\text{grav}})/E_{\text{imp}}$ MR=1,  $\theta=30$  $(M_{\text{lar}} - M_{\text{tar}})/M_{\text{pro}}$ MR=1,  $\theta=45$  $(M_{\text{lar}} - M_{\text{tar}})/M_{\text{pro}}$ MR=1,  $\theta=60$  $(M_{\text{lar}} - M_{\text{tar}})/M_{\text{pro}}$

# Dependence of resolution

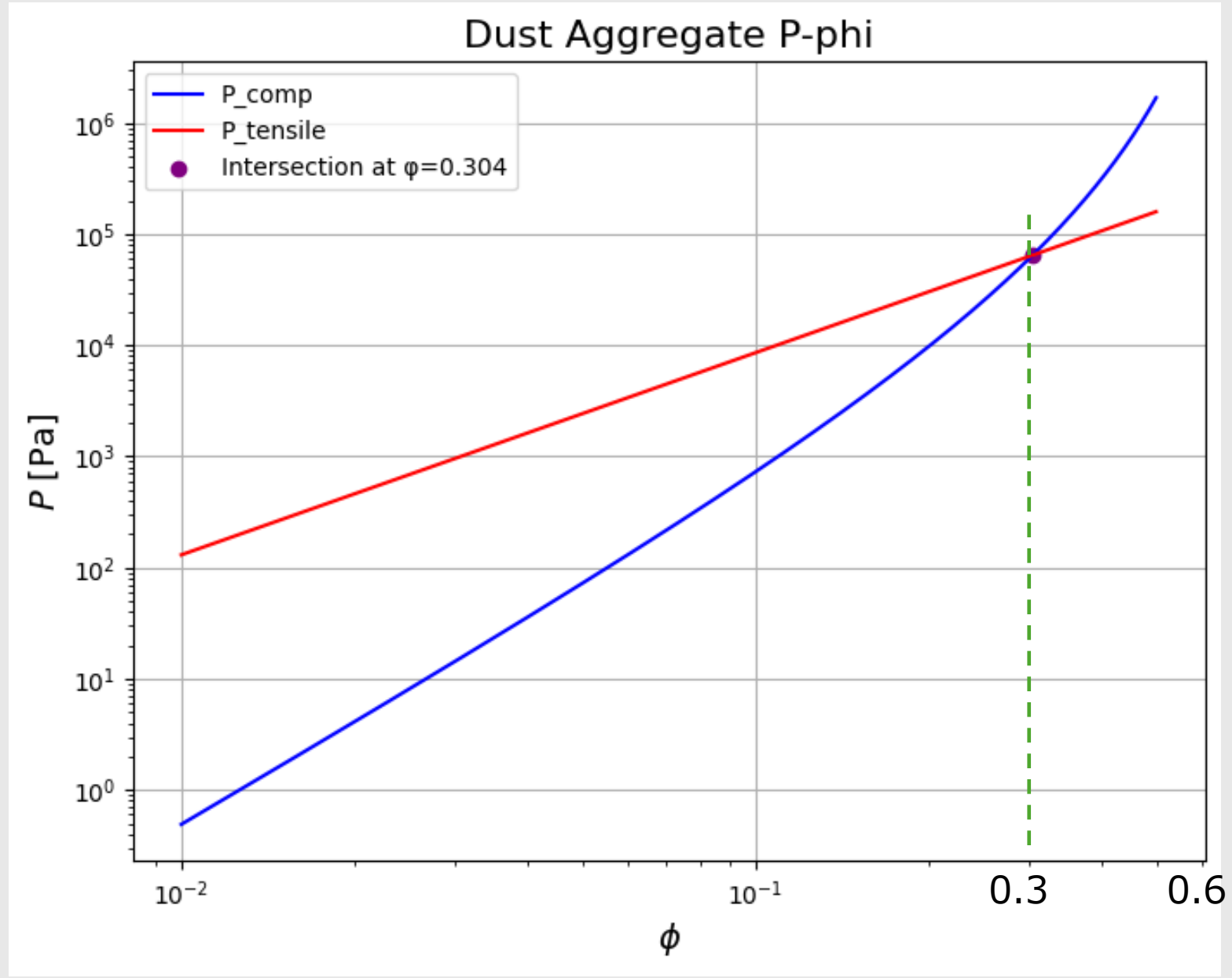
17



# Sound speed VS inverse of density



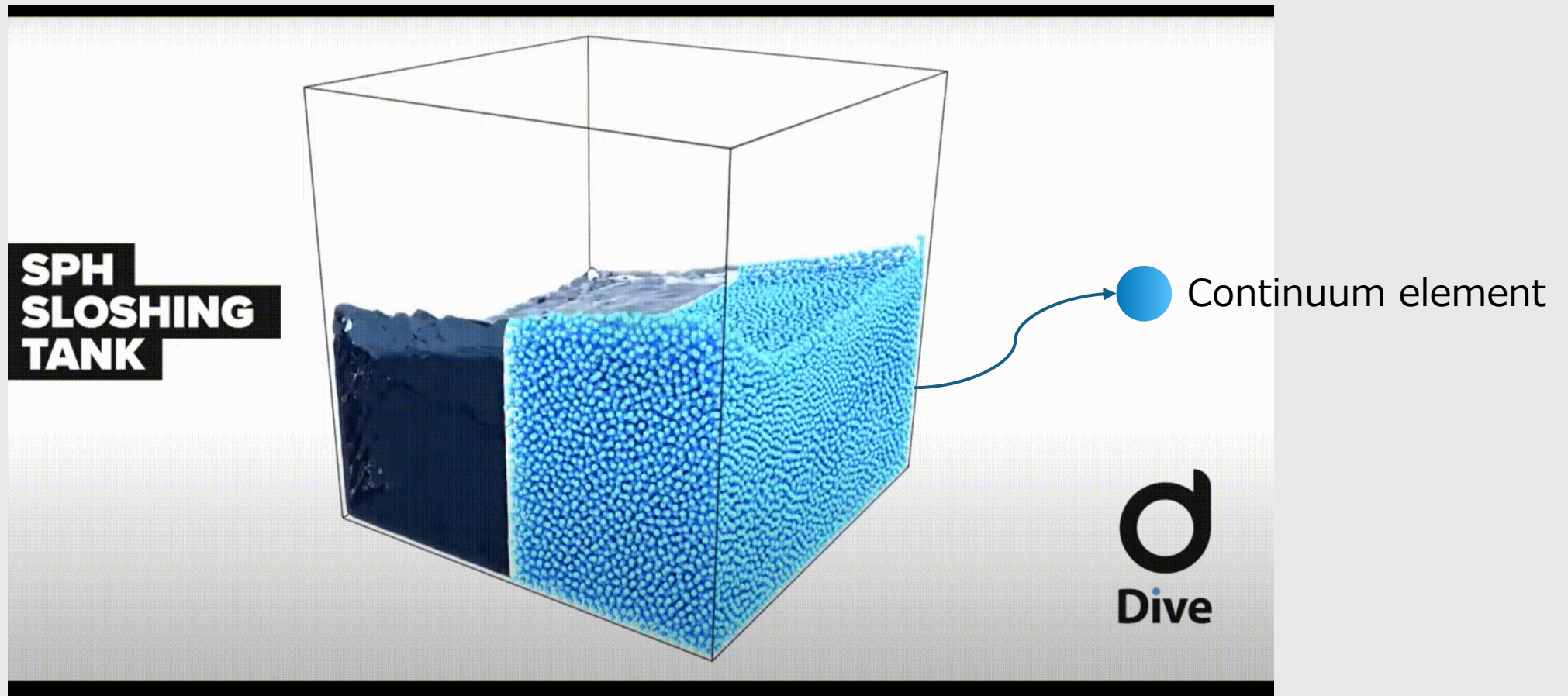
# Pressure VS inverse of density



# Appendix for SPH

# Method Fundamental concepts of the SPH method

1. SPH method is a particle method.



2. Physical quantities are represented a convolution with a kernel function.

Definition of density in SPH method :

$$\rho(x) \equiv \sum_j m_j W(x - x_j, h)$$

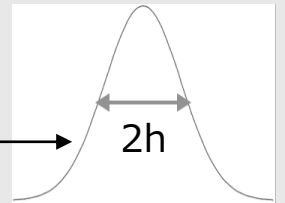
(  $m_j$  : Particle number in j-th SPH particle )

Definition of physical quantities in SPH method :

$$f(x) \equiv \int f(x) W(x - x_j, h) dx$$

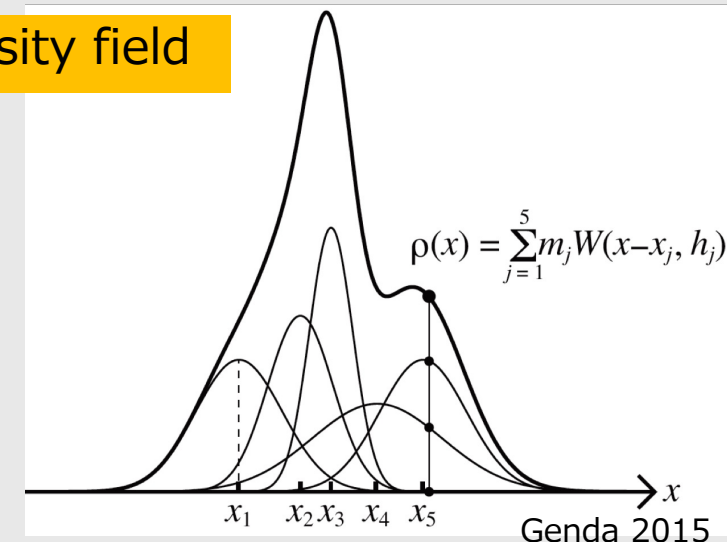
Kernel function :

$$W(x, h) \equiv \left( \frac{1}{h\sqrt{\pi}} \right)^d \exp\left( -\frac{x^2}{h^2} \right)$$



h: Smoothing length

Image of density field



## 微惑星の衝突速度

ガス円盤が残っている初期（微粒子～小さい微惑星）

乱流・ガス抗力で相対速度は  $m/s \sim 数十 m/s$  程度。

モデル計算では  $1 m \sim 1 km$  級でも多くが  $20-200 m/s$  程度にとどまるケースが示されています。

微惑星 ( $km \sim 100 km$ ) 同士・胚天体に攪乱される時期（走away/寡占成長）

典型的には脱出速度  $v_{esc}$  のオーダー ( $\geq v_{esc}$ )。

つまり  $数 \times 10 - 数 \times 10^2 m/s$  (対象サイズによる)

状況が進むと  $\sim 1 km/s$  未満  $\sim 数 km/s$  に達することも。

理論・数値では「実際の衝突は  $v_{imp} \geq v_{esc}$ 」が一般的とされています。

## 微惑星の衝突角度

ランダムな遭遇（等方分布）を仮定すると、幾何学的確率から衝突角度  $\theta$  の分布は

$$P(\theta) \propto \sin(2\theta), \quad 0 \leq \theta \leq 90^\circ$$

となる (Shoemaker 1962; Shoemaker 1969 など)。

この分布は、 $45^\circ$  付近の衝突が最も起こりやすいことを意味する。

垂直衝突 ( $0^\circ$ ) や極端なかすり衝突 ( $90^\circ$ ) は確率的には少なく、

$30-60^\circ$  の「斜め衝突」が全体の大部分を占める。

惑星形成において、角度のついた衝突はむしろ支配的で、 $0^\circ$  に近い正面衝突は少数派。

幾何学的に導かれる分布は で、 $45^\circ$  付近が最頻角度。

実際のN体シミュレーションでもほぼこの分布を示しており、斜め衝突を考慮することは必須。

# 既存のエネルギー議論

Specific impact energy  
比衝突エネルギー

$$Q = \frac{1}{2} \frac{\mu v_{imp}^2}{M_{tot}} \quad \mu = \frac{M_{tar} M_{imp}}{M_{tar} + M_{imp}} \quad Q = \frac{E_{imp}}{M_{tar} + M_{imp}}$$

衝突エネルギーの定義:  
単位質量あたり、  
どれだけの衝突エネルギーが入るか  
ある天体を半分以上破壊するために必要な比衝突エネルギー

Disruption energy  
破壊エネルギー

$$Q_{d*} = \text{(材料強度)} + \text{(重力束縛)}$$

↑ カチカチの岩石で単純モデル化

強度支配領域 $D < \text{数km}$	重力支配領域 $D > \text{数10km}$
----------------------------	------------------------------

破壊エネルギーの定義:  
ある天体を半分以上破壊するために必要な比衝突エネルギー

**議論の典型**

もし  $Q_{d*} > Q$  なら破壊  
もし  $Q_{d*} < Q$  なら合体

## ◆ 問題点

- 衝突角度や質量比を無視
  - $Q$  の定義自体は質量比を入れているが、議論は主に「衝突速度」と「サイズ」だけに戻りがち。
  - 斜め衝突 (2回衝突 や hit-and-run) などは表現できない。
- 散逸過程を無視
  - 摩擦、塑性変形、部分破壊などは「単なるエネルギー収支」では表現できない。
- 実際の結果が連続的であることを無視
  - 実際には「完全合体」「完全破壊」の中間に hit-and-run, erosion, partial accretion が存在する。

→ これらを捉えるには 数値シミュレーション など で 詳細な物理をモデル化する必要がある。

