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Fukuhara, Flock, Okuzumi, & Tominaga, 2025

Hydrodynamical simulations of the vertical shear instability with dynamic dust and cooling rates in protoplanetary disks

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Initial stage of planet formation

✤ The stage of planet formation is the protoplanetary disk

Protoplanetary disk

~ 100 au

Gas + Dust

young star

$$\frac{\textbf{Dust mass}}{\textbf{Gas mass}} \sim 0.01$$

Dust evolution / Planetesimal formation

Planet formation

Dust

Grain — $\sim \mu$m

Pebble — $\sim$ cm

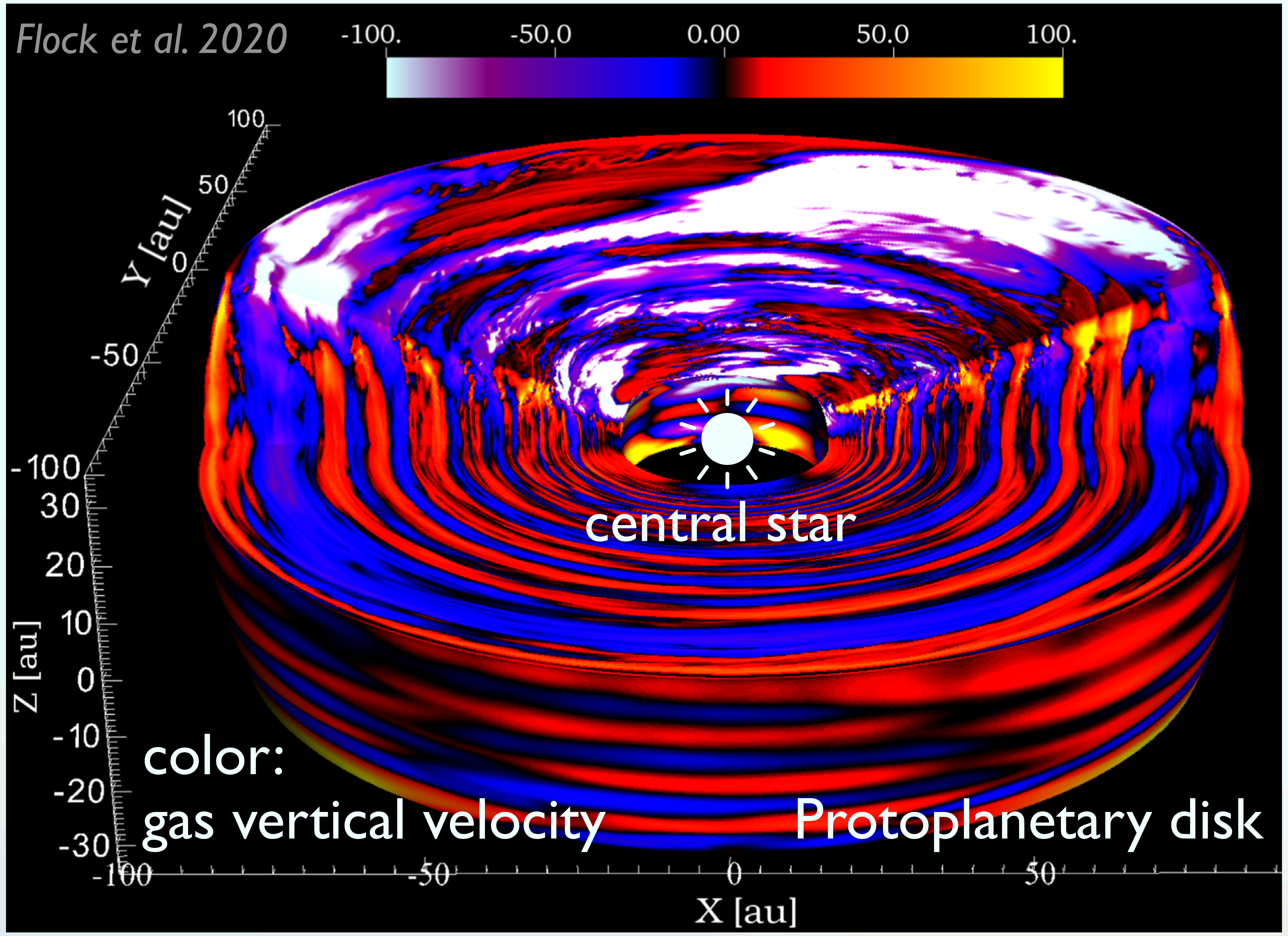
Planetesimal — $\sim$ km

Planet — $\sim 10^3$km

Turbulence in protoplanetary disks

- ✤ Protoplanetary disks can be **turbulent**
- ✤ Gas turbulence plays many roles on dust growth evolution

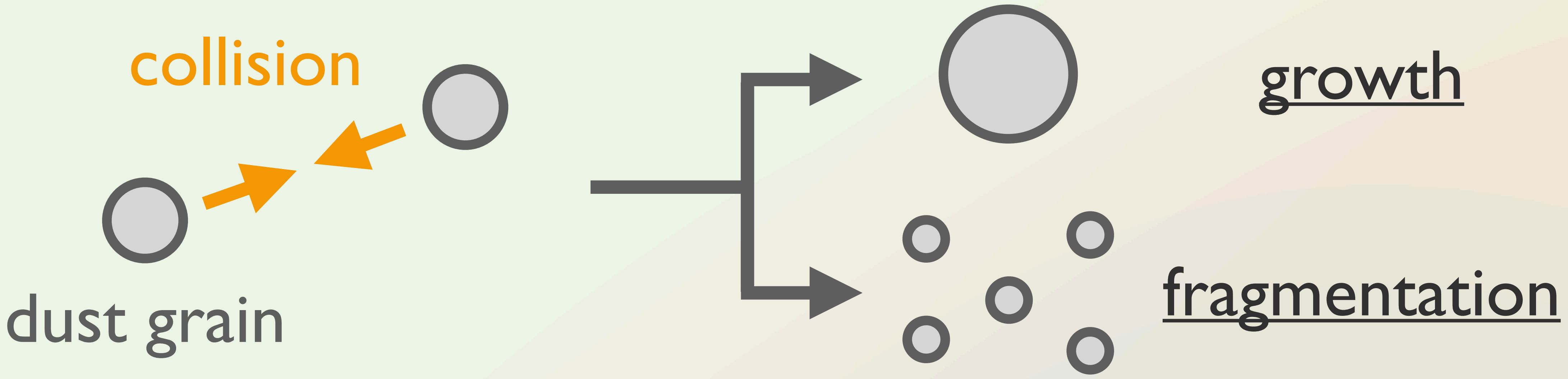
disk gas turbulence simulation



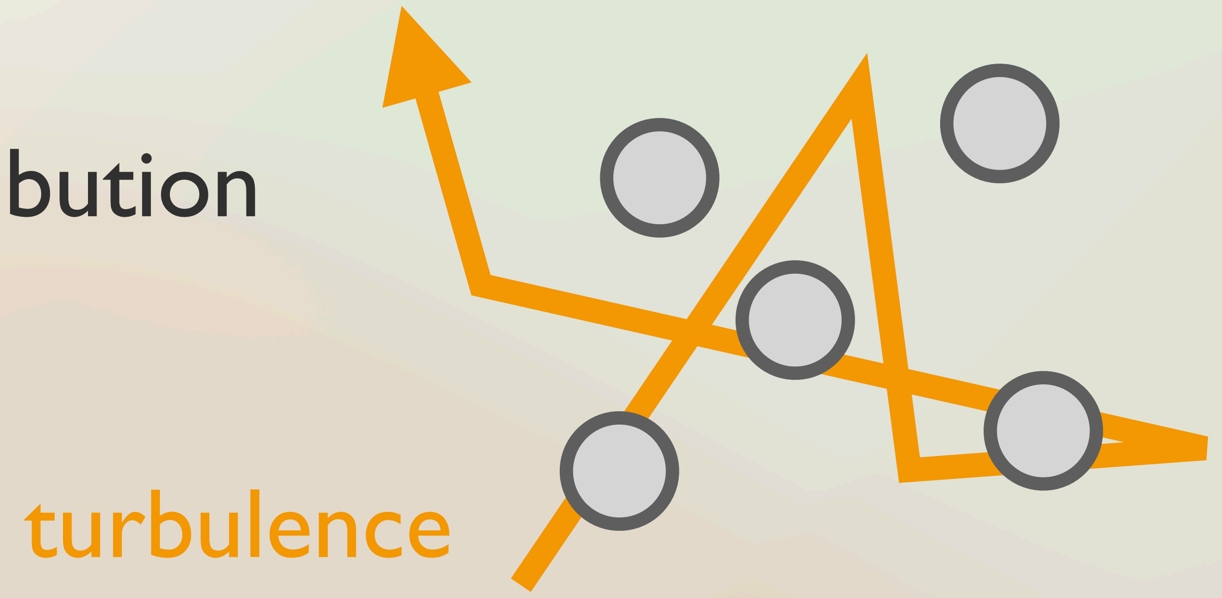
driving mechanisms

Dust

- induce collisional velocity between dust particles
  - determine dust size



- diffuse dust particle
  - determine dust distribution



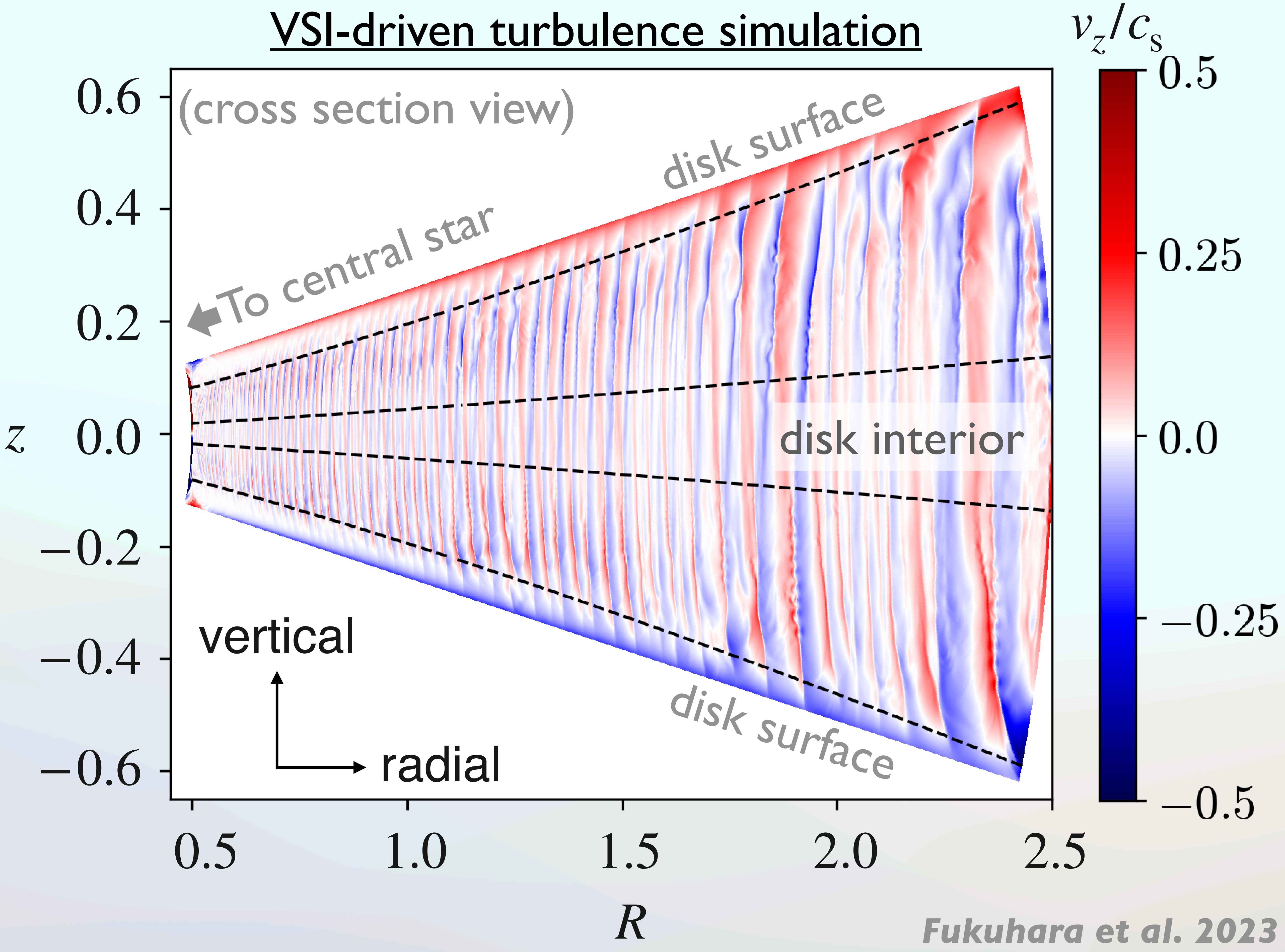
Vertical shear instability

✤ One of the candidate mechanisms driving turbulence in disks is **vertical shear instability (VSI)**

for a review, Lesur et al. 2023

- Disk-version of the Goldreich-Schubert-Fricke instability *Goldreich & Schubert 1967; Fricke 1968*
- Purely (thermo-)hydrodynamical instability *Arlt & Urpin 2004; Nelson et al. 2013*

<ins>VSI-driven turbulence simulation</ins>



Fukuhara et al. 2023

✤ VSI generates turbulence with **predominant vertical motion**

e.g., Nelson et al. 2013; Stoll & Kley 2016

- VSI is driven by vertical gradient of orbital velocity

✤ VSI requires **rapid gas cooling** *e.g., Lin & Youdin 2015*

- Rapid cooling can kill buoyancy that prevents VSI growth

Interaction between VSI and dust

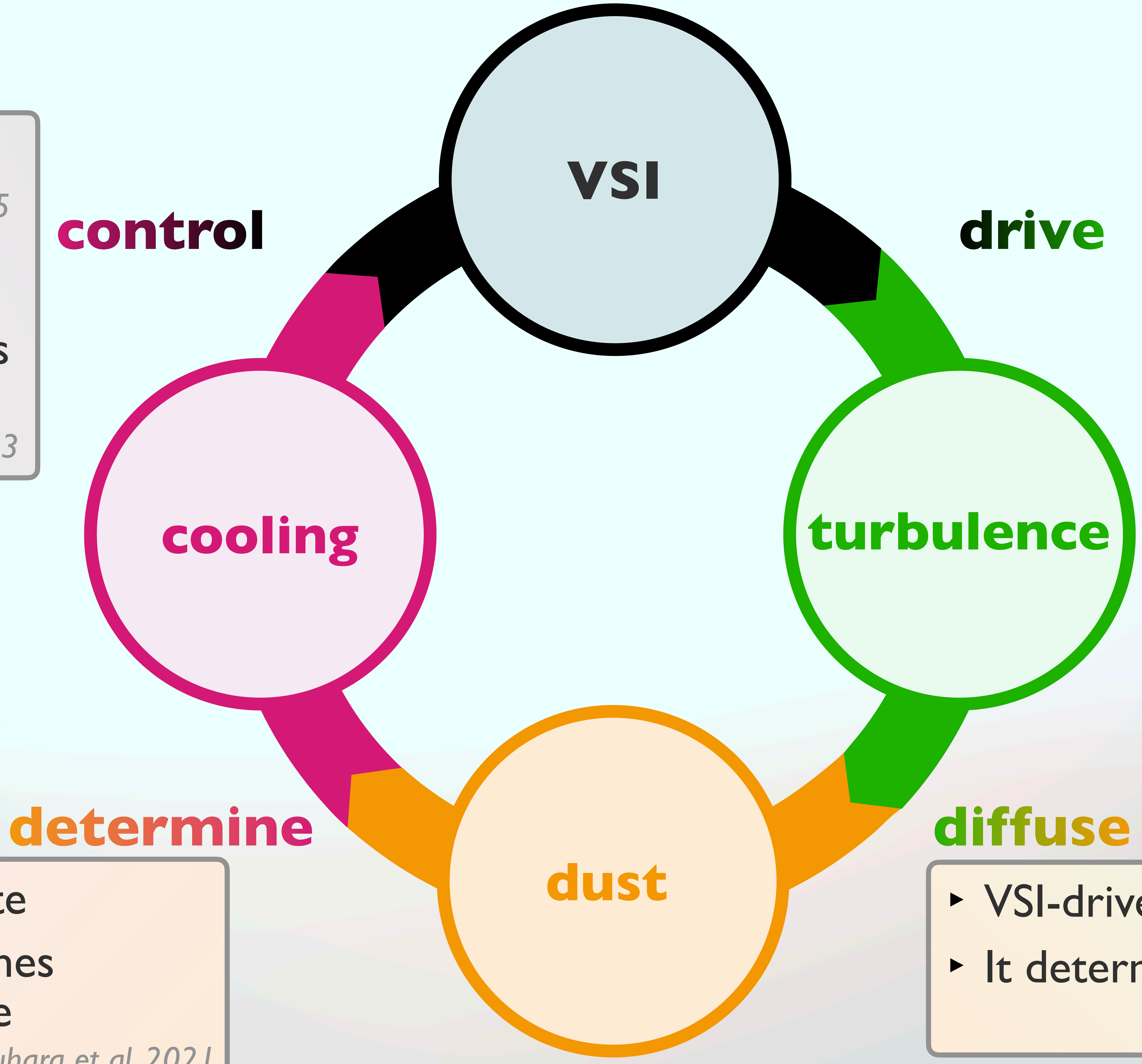
interaction between VSI and dust!

- Cooling rate controls VSI's onset *e.g., Lin & Youdin 2015*

$$t_{\rm cool}\Omega_{\rm K} \lesssim 0.1$$

- Cooling rate profile controls turbulence intensity

e.g., Fukuhara et al. 2023



- VSI drives turbulence with predominant vertical motion

e.g., Nelson et al. 2013; Stoll & Kley 2014

- Dust determines cooling rate
- Dust spatial profile determines spatial profile of cooling rate

e.g., Pfeil & Klahr 2019; Fukuhara et al. 2021

- VSI-driven turbulence diffuses dust
- It determines dust spatial profile

e.g., Flock et al. 2020; Dullemond et al. 2022

Purpose of this study

- Our previous studies have shown that…
  - VSI-active region depends on dust grain size and spatial profile **Fukuhara et al. 2021**
  - VSI-driven turbulence depends on the dust vertical profile **Fukuhara et al. 2023**
  - There can exist the equilibrium state where settling balances with turbulence diffusion **Fukuhara & Okuzumi 2024**

- Questions

Can VSI drive turbulence in dynamical cooling rate?

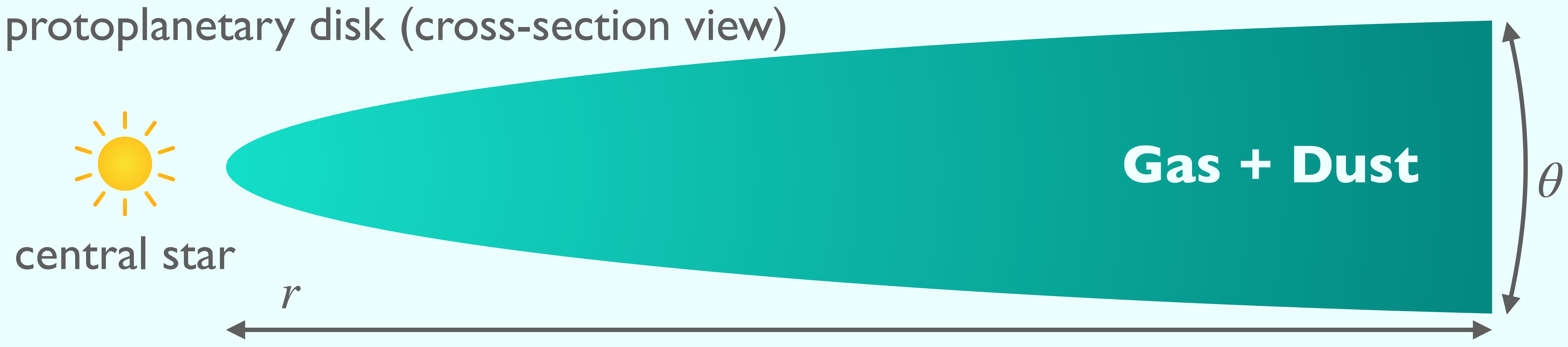
How is dust spatial profile determined in VSI-driven turbulent disks?

Does there exist equilibrium for turbulence and dust profile?

Method: simulations including gas and dust

✤ We perform **2.5D hydrodynamical simulations** of an axisymmetric protoplanetary disk

protoplanetary disk (cross-section view)



- radial-meridional $(r,\ \theta)$ plain
  + three components for velocities $(v_r,\ v_\theta,\ v_\phi)$
- Code: PLUTO + dust module
  Mignone et al. 2007; Ziampras et al. 2025
- resolution $\approx 70\ \mathrm{cells/H_{gas}}$

gas

$$\frac{\partial \rho_{\rm gas}}{\partial t} + \nabla \cdot (\rho_{\rm gas}\boldsymbol{v}_{\bf gas}) = 0$$

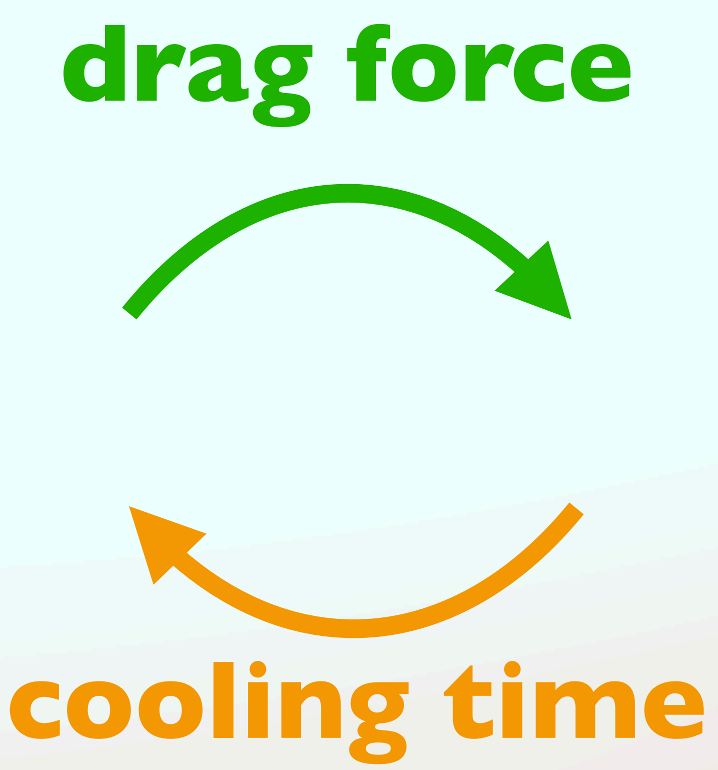
$$\frac{\partial \rho_{\rm gas}\boldsymbol{v}_{\bf gas}}{\partial t} + \nabla \cdot \left(\rho_{\rm gas}\boldsymbol{v}_{\bf gas}\boldsymbol{v}_{\bf gas}{}^{T}\right) = -\nabla P - \rho_{\rm gas}\nabla\Phi$$

$$\frac{\partial E_{\rm t}}{\partial t} + \nabla \cdot [(E_{\rm t} + P)\boldsymbol{v}_{\bf gas}] = -\rho_{\rm gas}\boldsymbol{v}_{\bf gas} \cdot \nabla\Phi$$

$$\boxed{\frac{dP}{dt} = -\frac{P - P_{\rm ini}}{t_{\rm cool}}}$$

cooling

e.g., Manger et al. 2020



drag force

cooling time

dust

- single size dust as pressureless fluid
  - dust grain size: a

$$\frac{\partial \rho_{\rm dust}}{\partial t} + \nabla \cdot (\rho_{\rm dust}\boldsymbol{v}_{\bf dust}) = 0$$

$$\frac{\partial \rho_{\rm dust}\boldsymbol{v}_{\bf dust}}{\partial t} + \nabla \cdot \left(\rho_{\rm dust}\boldsymbol{v}_{\bf dust}\boldsymbol{v}_{\bf dust}{}^{T}\right) = -\rho_{\rm dust}\nabla\Phi + \boxed{\rho_{\rm dust}\frac{\boldsymbol{v}_{\bf gas} - \boldsymbol{v}_{\bf dust}}{t_{\rm stop}}}$$

drag force

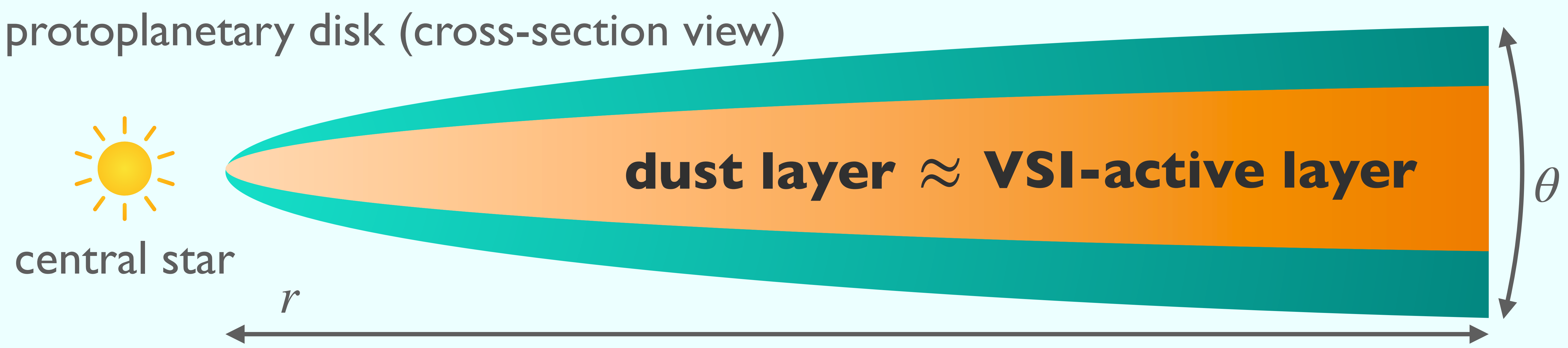
$t_{\rm stop} = \dfrac{\rho_{\rm int}a}{\rho_{\rm gas}c_{\rm s}}$: stopping time

$\rho_{\rm int} = 1.46\ \mathrm{g\ cm^{-3}}$: dust internal density

ρ :density $\boldsymbol{v}$:velocity P :puressure Φ :gravitational potential of central star E_t :the total energy per unit volume $t_{\rm cool}$:cooling time

Method: simulations including gas and dust

✤ We perform **2.5D hydrodynamical simulations** of an axisymmetric protoplanetary disk



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gas

- cooling time depends on dust
  - assuming optically thin regime
  - main process is collisional heat transfer

  Malygin et al. 2017; Pfeil & Klahr 2019; Fukuhara et al. 2021

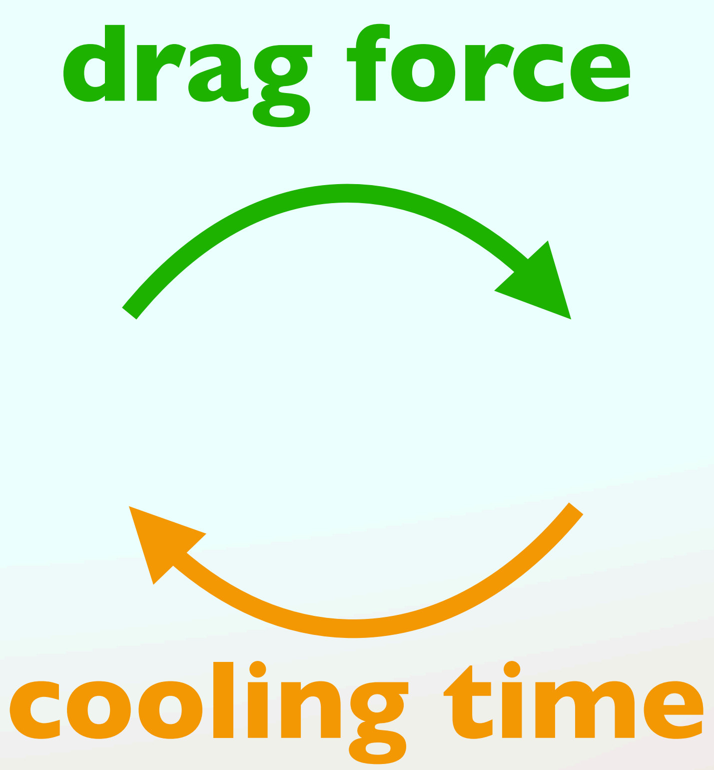
$$t_{\mathrm{cool}} = \frac{l_{\mathrm{gd}}}{v_{\mathrm{th}}}$$

$$l_{\mathrm{gd}} = \left(\pi a^2 \cdot \frac{\rho_{\mathrm{dust}}}{m_{\mathrm{dust}}}\right)^{-1} \propto \left(\frac{a}{\rho_{\mathrm{dust}}}\right)$$

dust size

dust density

- VSI criterion $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \lesssim 0.1$ *e.g., Lin&Youdin 2015*



dust

- single size dust as pressureless fluid
  - dust grain size: a
- motion is determined by **drag force**
- main parameters
  - **dust grain size**
    $10\ \mu\mathrm{m},\ 30\ \mu\mathrm{m},\ 100\ \mu\mathrm{m},\ 300\ \mu\mathrm{m}$
  - **global dust-to-gas mass ratio**
    $0.001,\ 0.003,\ 0.01,\ 0.03,\ 0.1$

l_{gd} : mean travel length of gas molecules colliding with dust particles v_{th} : thermal velocity m_{dust} : dust grain mass

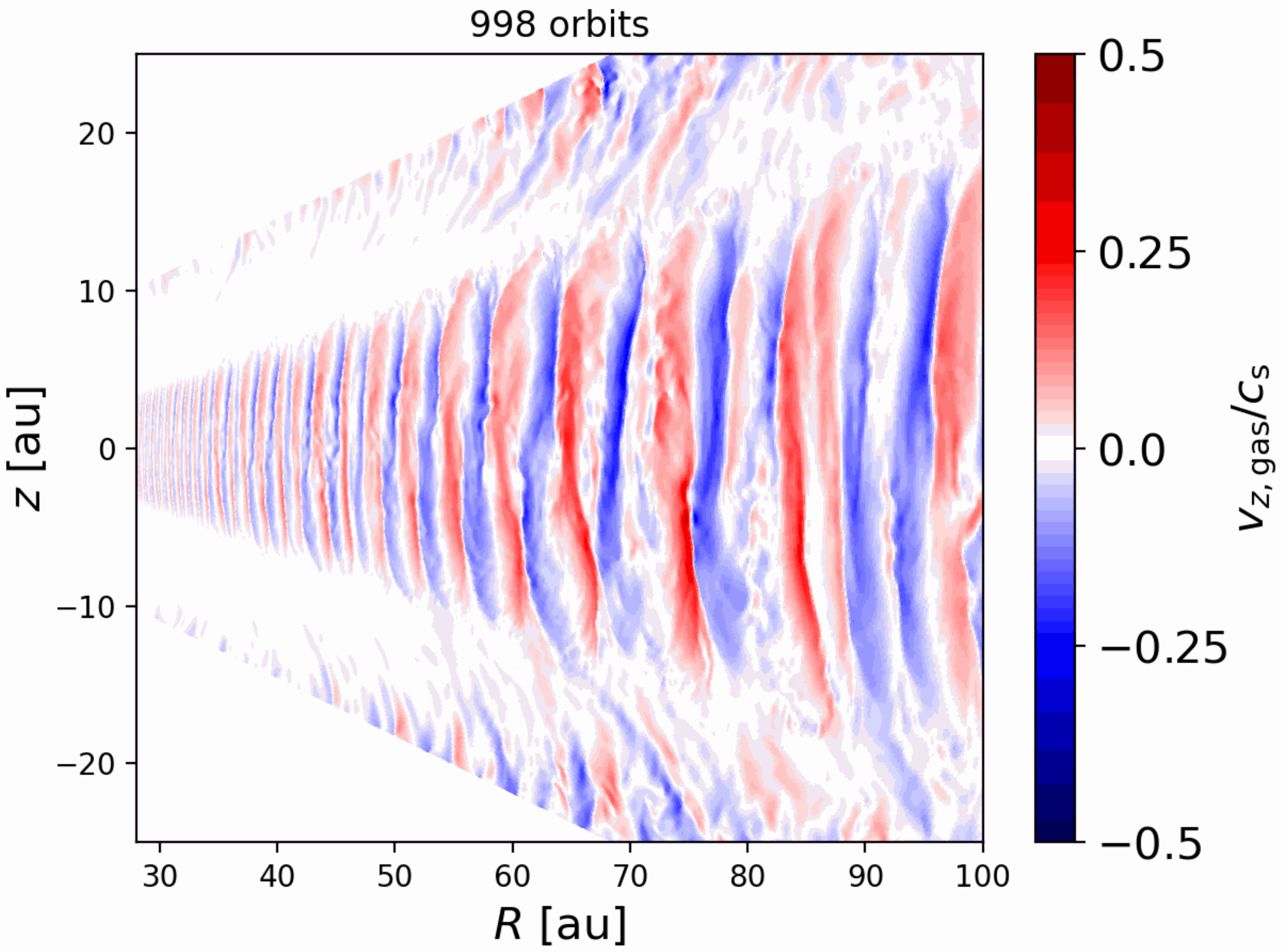
Results: equilibrium state for turbulence and dust

- ✤ There exists the **equilibrium state** for VSI turbulence and dust profile
- ✤ VSI can generate turbulence that diffuses dust in the vertical direction

grain size = $10\ \mu m$

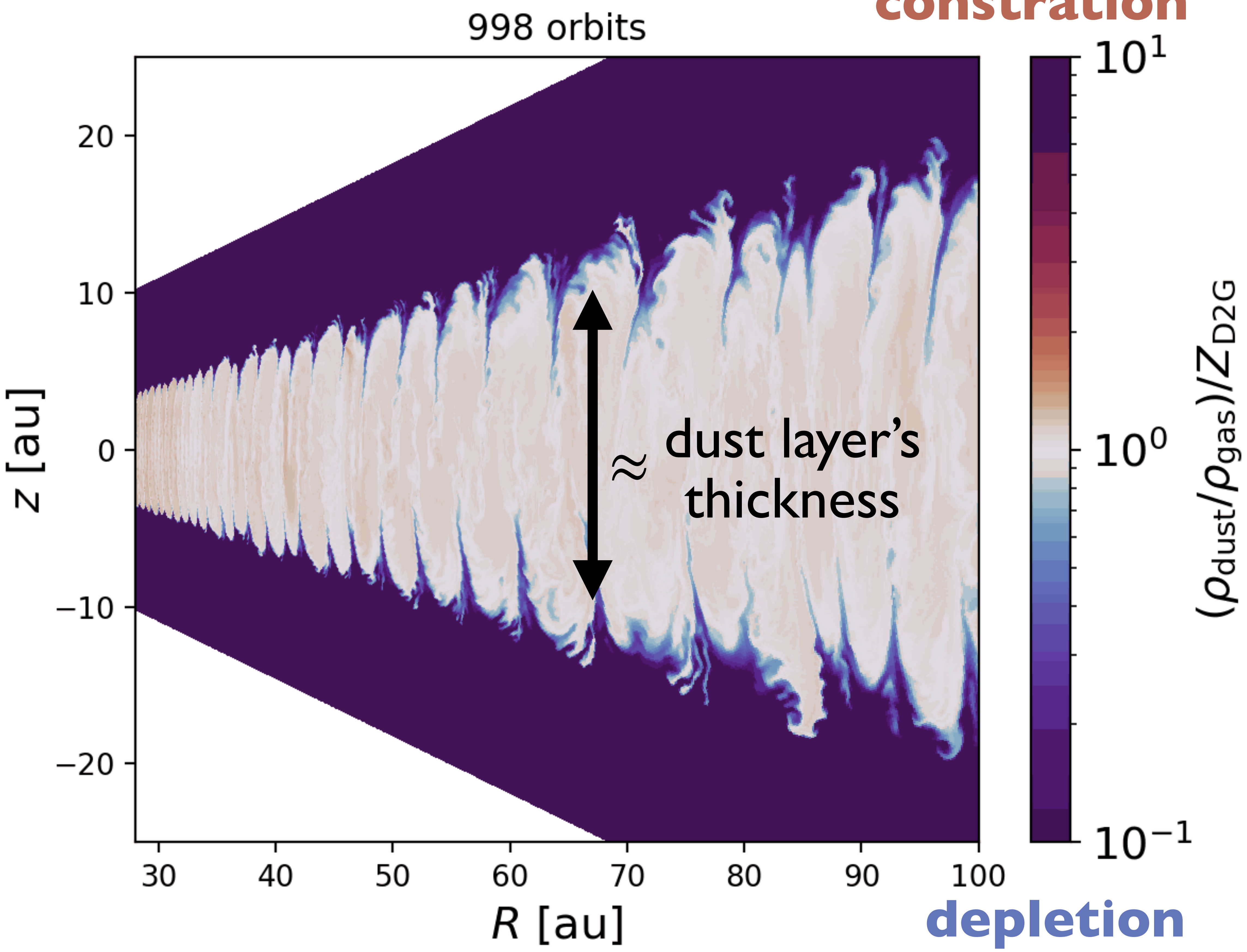
dust-to-gas ratio = 0.01

gas vertical velocity $v_{z,\mathrm{gas}}$



dust-to-gas ratio / initial dust-to-gas ratio

constration



depletion

Results: vertical diffusion coefficient

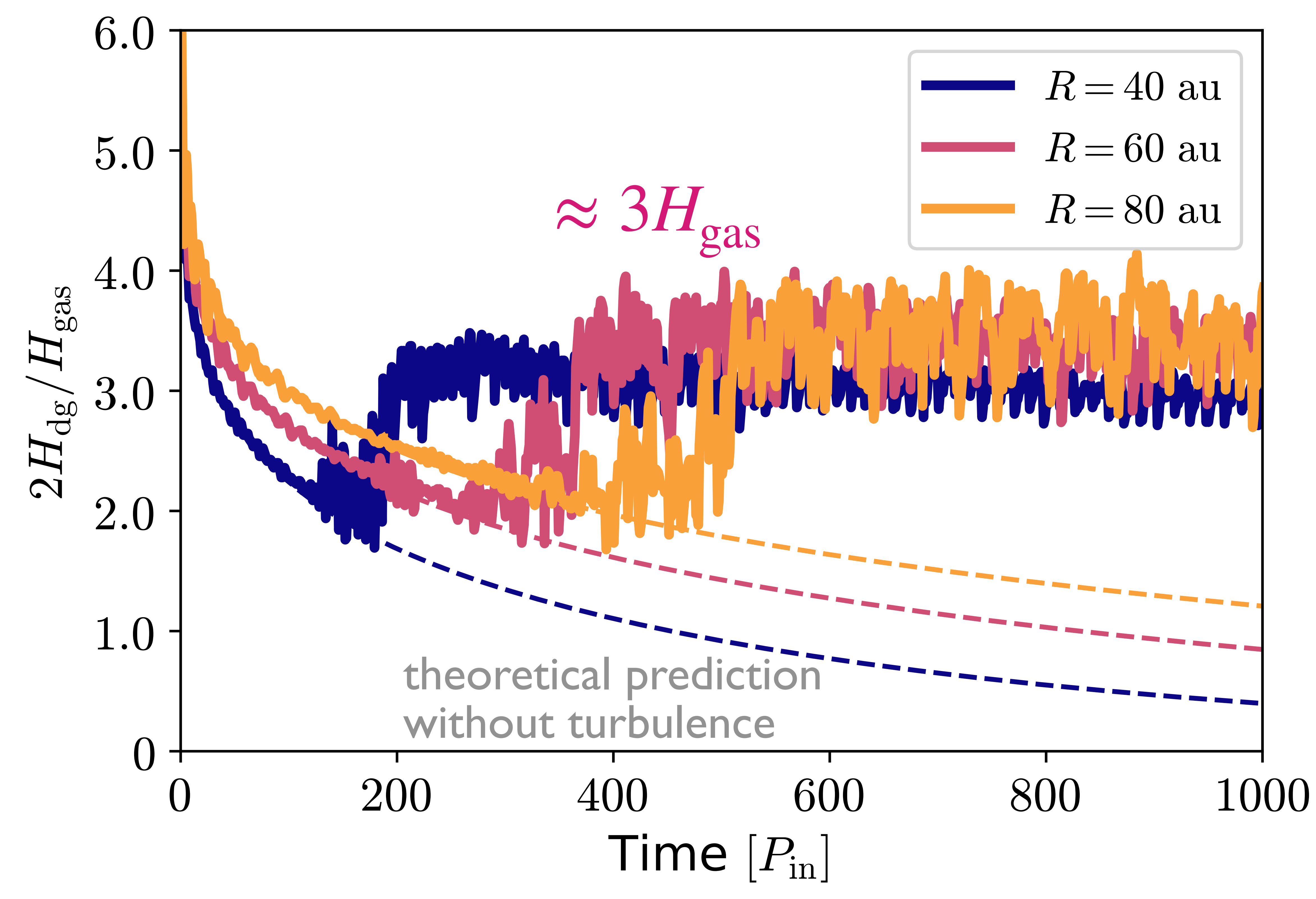
✤ We estimate vertical diffusion coefficient from dust layer's thickness

vertical profile of $\rho_{\rm dust}/\rho_{\rm gas}$ → dust layer's thickness $H_{\rm dg}$ → $H_{\rm dg} = \left(\frac{\rm St}{\alpha_{\rm diff}}\right)^{-1/2} H_{\rm gas}$ → **dust vertical diffusion coefficient** $\alpha_{\rm diff}$

$\mathrm{St} = t_{\rm stop}\Omega_{\rm K} \sim 10^{-4}$

Dubrulle et al. 1995

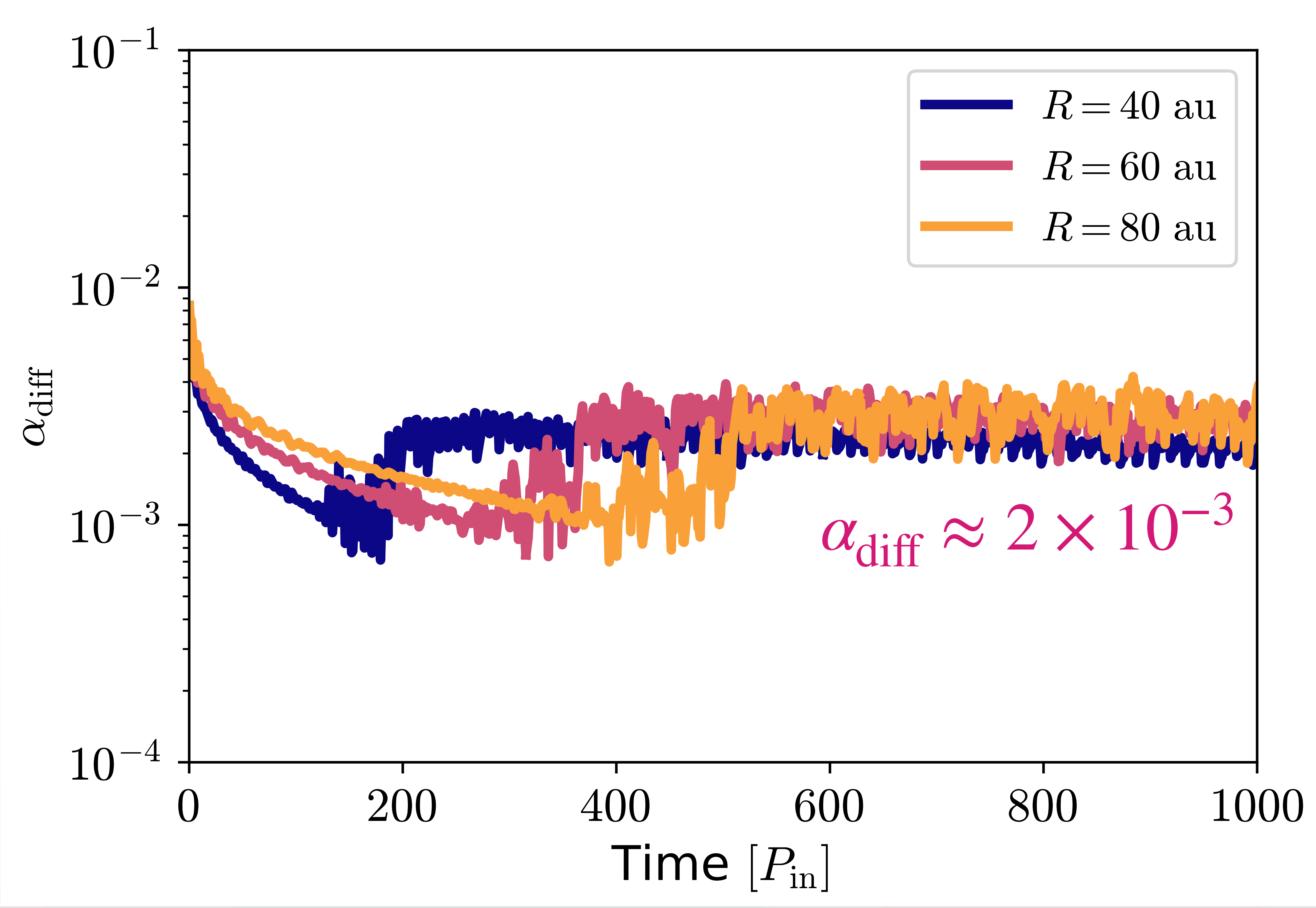
time evolution of dust layer's thickness



$\approx 3H_{\rm gas}$

theoretical prediction
without turbulence

time evolution of dimensional vertical diffusion coefficient



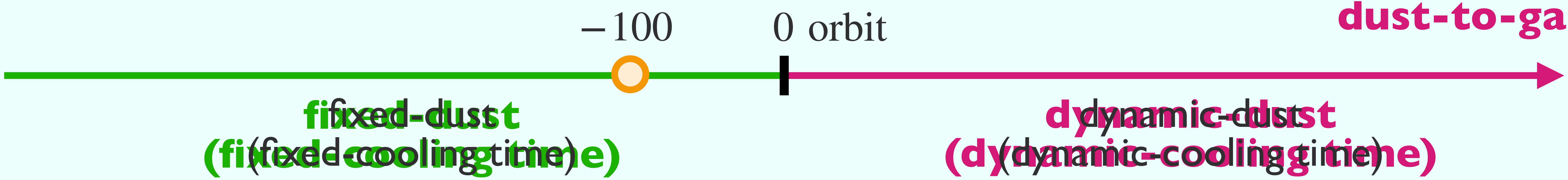
$\alpha_{\rm diff} \approx 2 \times 10^{-3}$

Results: independence of initial condition

- ✤ This equilibrium does not depend on the initial condition!

grain size = $10\ \mu m$

dust-to-gas ratio = 0.01


-100

0 orbit

fixed-dust
(fixed-cooling time)

fixed-dust
(fixed-cooling time)

dynamic-dust
(dynamic-cooling time)

dynamic-dust
(dynamic-cooling time)


gas vertical velocity $v_{z,\mathrm{gas}}$

dust-to-gas ratio / initial dust-to-gas ratio

constration

depletion

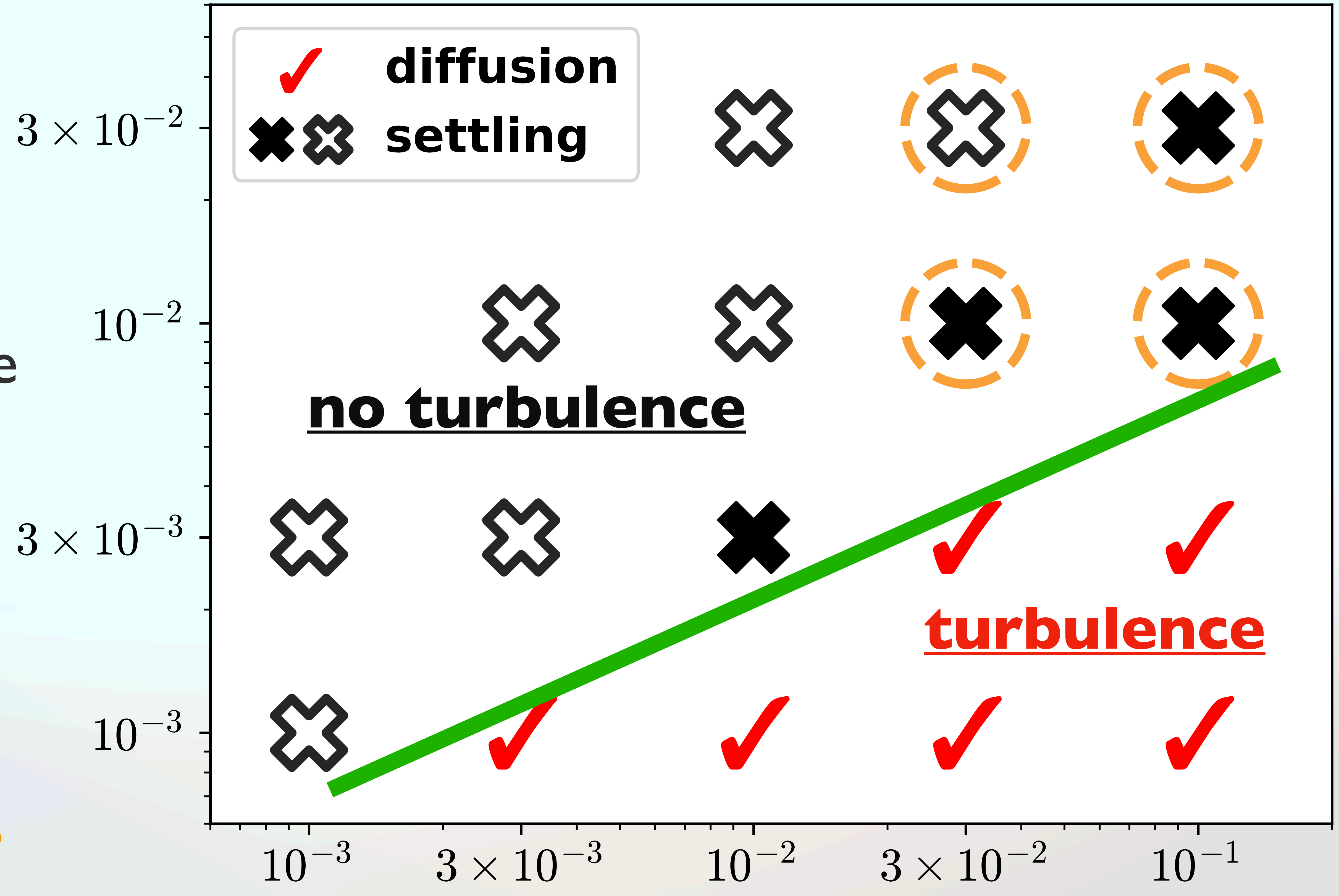
Results: dependence of grain size and total dust mass

✤ Equilibrium states can exist only for **small dust grains** or **high dust-to-gas mass ratio**

- large dust grains
- low dust-to-gas mass ratio

→ VSI does not drive turbulence, and dust settles toward the midplane

Summary of our simulations



◌ : $\rho_\mathrm{dust}/\rho_\mathrm{gas} \geq 1$ at the midplane

dust grain size
a [cm]

↓ **small grains**

dust-to-gas mass ratio → **high dust-to-gas mass ratio**

consistent with previous prediction

Fukuhara & Okuzumi 2024

↓

This is determined by balance between diffusion and settling

Discussion: link to planet formation

- ✤ We find that the onset of VSI-driven turbulence depends on **dust grain size**

turbulence and dust co-evolve!

protoplanetary disks

criterion?

VSI strong turbulence

dust collisional growth

collision

very weak turbulence

dust growth via gravitational collapse

$1\ \mu$m $10\ \mu$m $100\ \mu$m 1 mm grain size

dust growth in constant-intensity turbulent disks

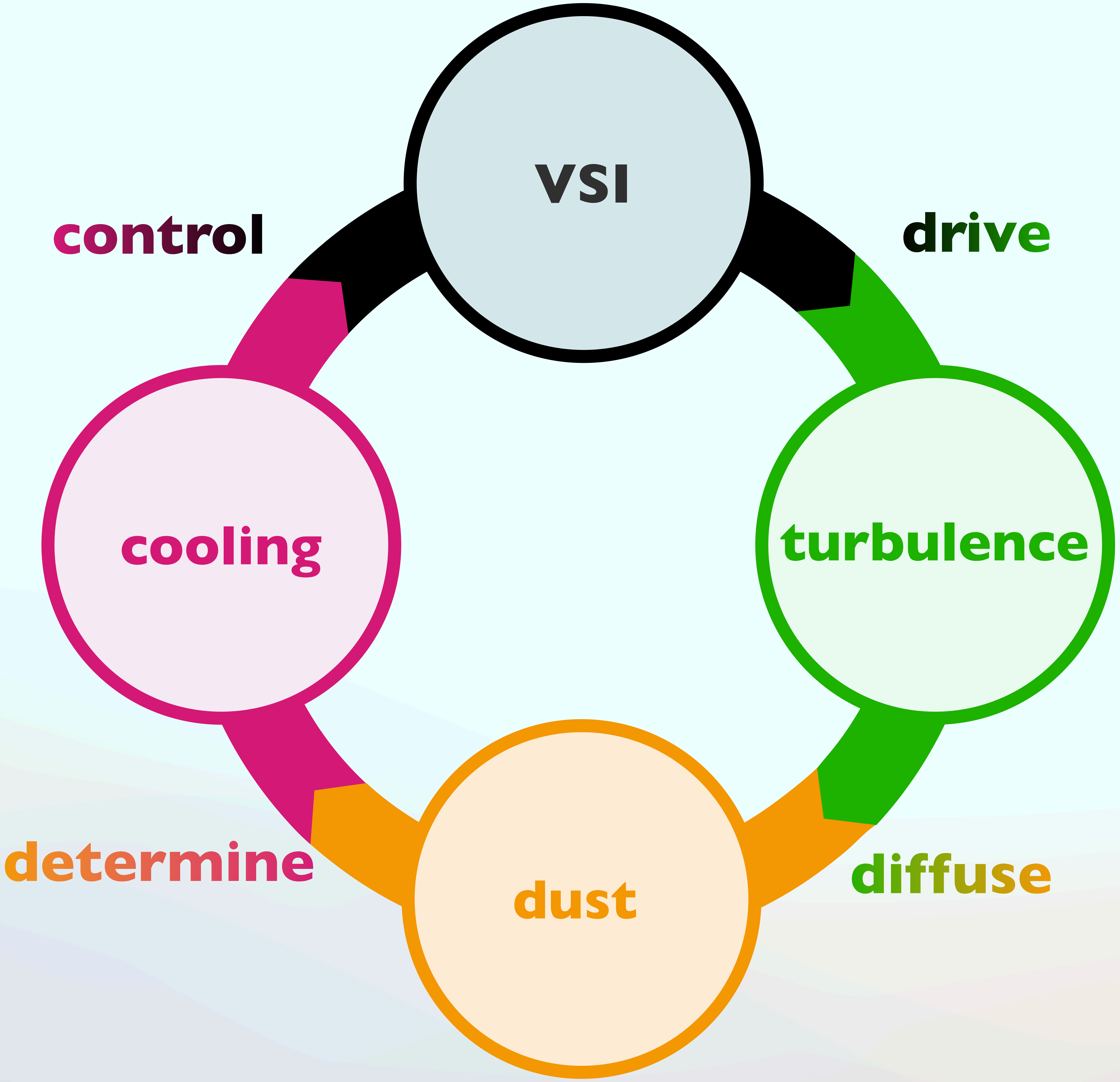
strong/weak turbulence

collisional growth

dust grain

$1\ \mu$m $10\ \mu$m $100\ \mu$m 1 mm grain size

Summary & Take home message



✤ Purpose

- To understand co-evolution between VSI turbulence, dust, and cooling rate

✤ Method

- 2.5D hydrodynamical simulations of an axisymmetric disk with dynamic dust and cooling rate

✤ Result

- There exist equilibrium state for VSI turbulence and dust profile
- The onset depends on grain size and total dust mass

✤ Take home message

VSI turbulence and dust coevolve!