



清华大学天文系
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MHD-gPIC Method for Multiscale Plasma Kinetic Simulations

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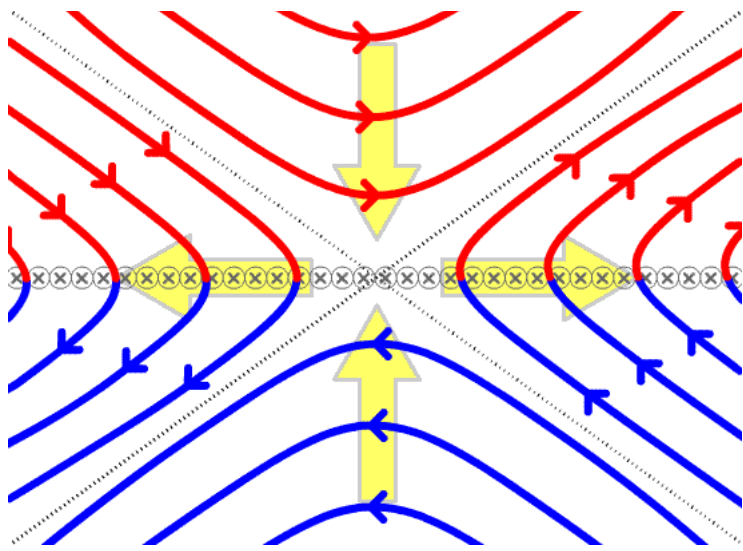
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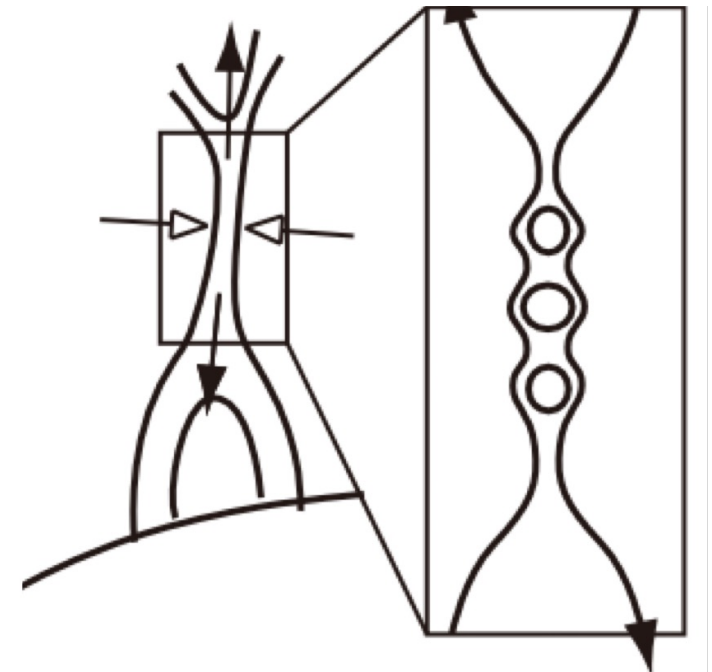
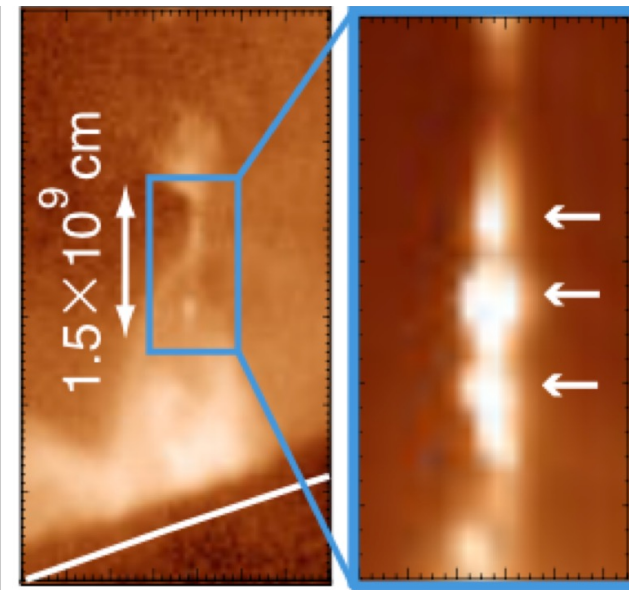
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Reconnection as a high-energy particle source

- **Magnetic reconnection:** a fundamental process in plasma physics and astrophysics
- Occurs when magnetic field lines rearrange and reconnect, leading to energy release
- Happens in various settings: Earth's magnetosphere, solar flares, pulsar magnetosphere, etc.



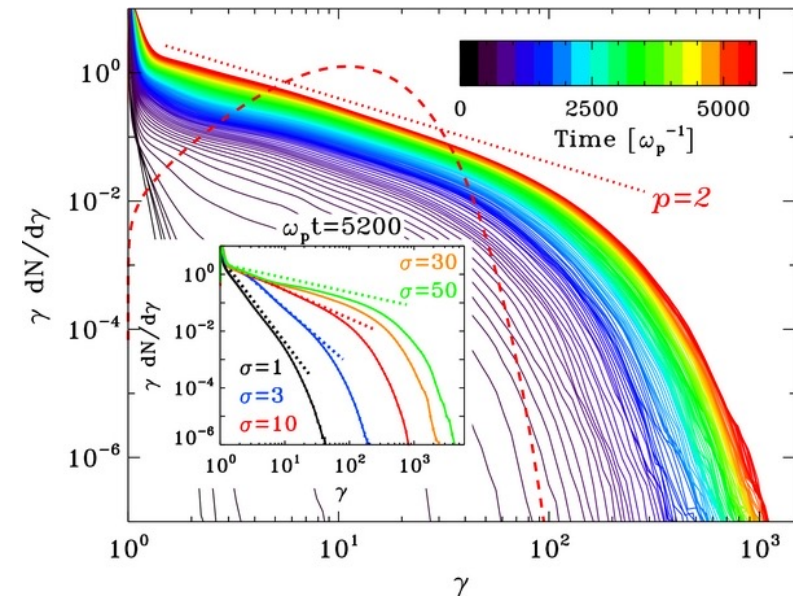
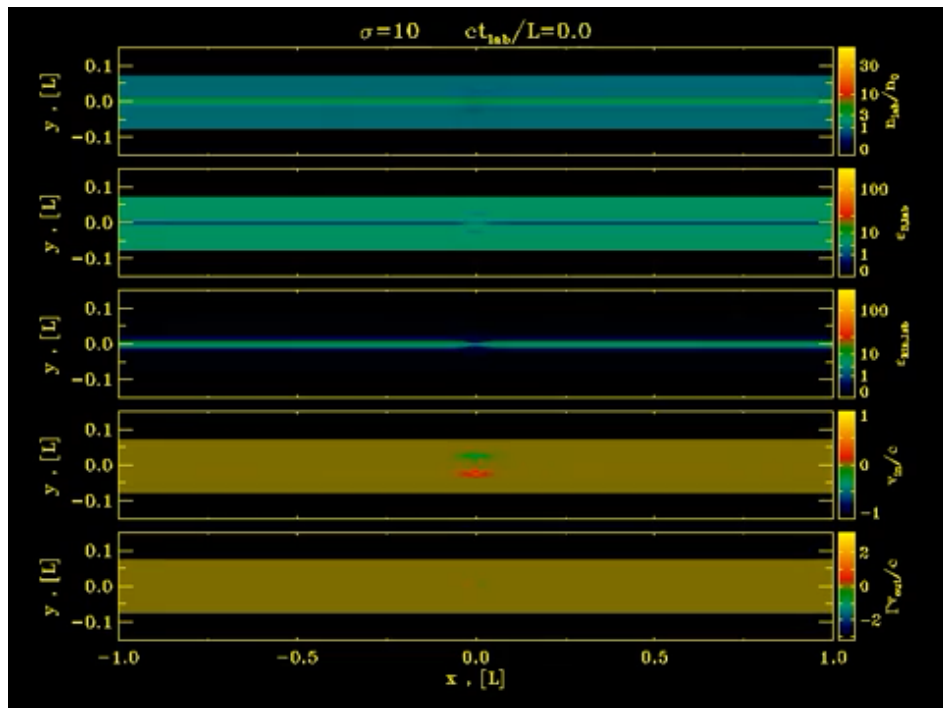
Conversion of magnetic energy into heat and energetic particles (up to ~50% energy).



Takasao et. al., 2012

Numerical studies of magnetic reconnection

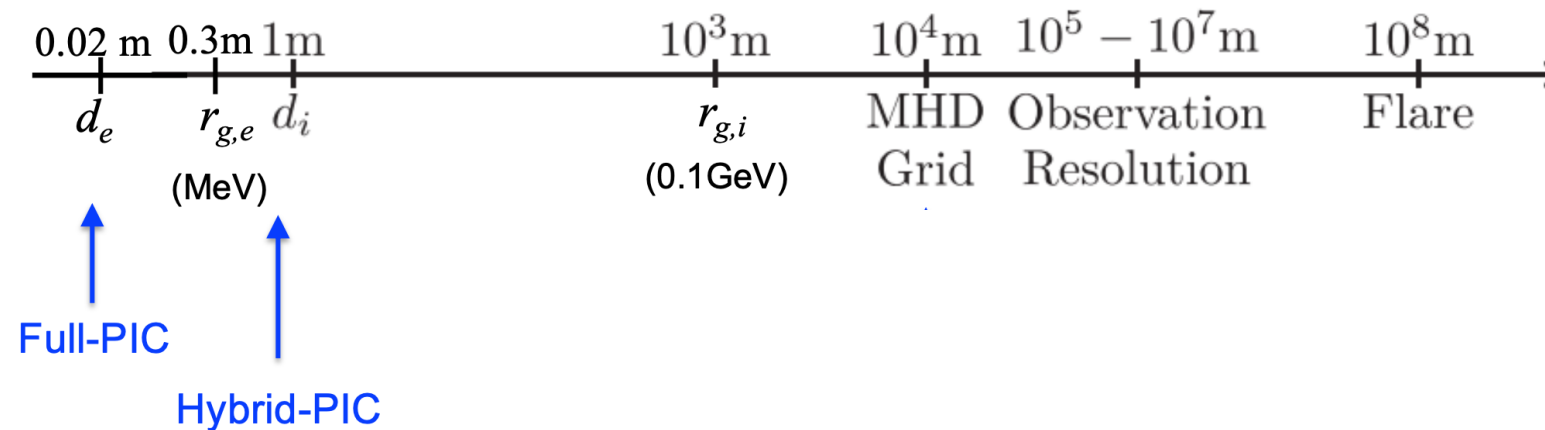
- PIC simulations have become routine in studying relativistic reconnection (e.g., Guo et al. 2014)
- Production of non-thermal particle population with a very hard spectrum.



Sironi & Spitkovsky, 2014

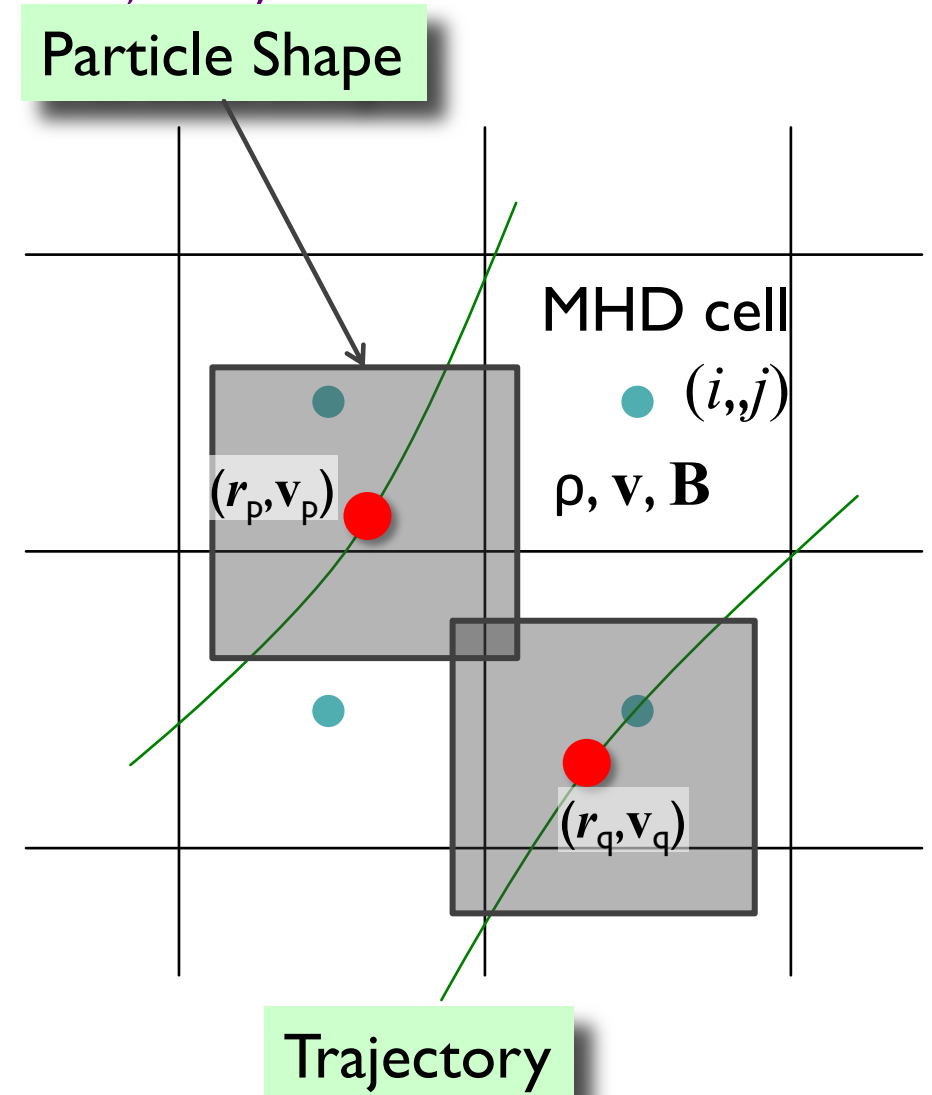
Issue with scale separation

- Dramatic scale separation involved in solar reconnection problem with particle acceleration.



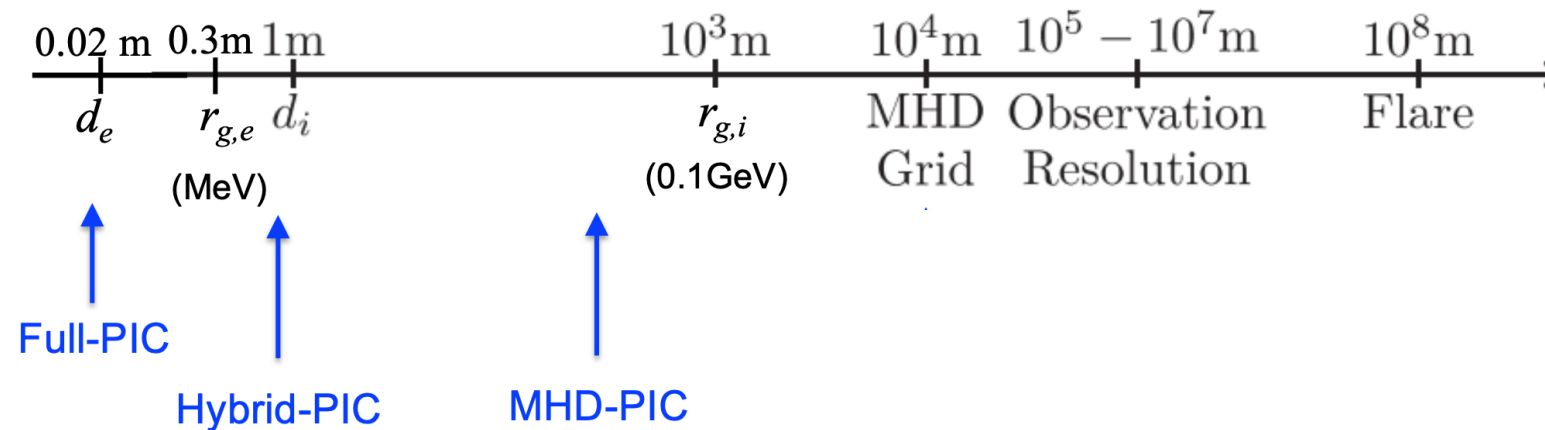
MHD-PIC approach (Bai, et. al., 2015; Sun & Bai, 2023)

- Each computational particle (i.e., **super-particle**) represents a large collection of real particles.
- Each super-particle carries an effective **shape**, designed to facilitate interpolation from the grid.
- Individual particles move under the electromagnetic field from MHD.
- Total momentum and energy must conserve: particles **feedback** to MHD by depositing changes in **momentum and energy locally**.



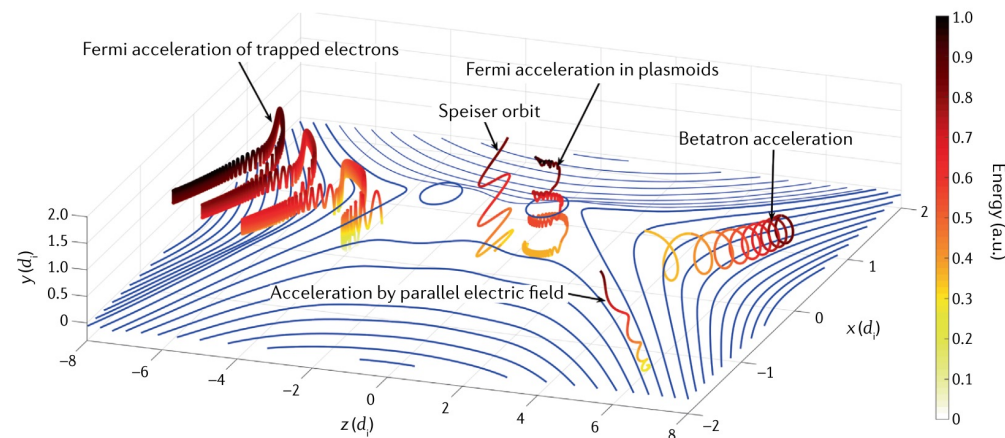
Issue with scale separation

- Dramatic scale separation involved in solar reconnection problem with particle acceleration.
- MHD-PIC: **treat background plasma by MHD, while non-thermal particles are kinetic** (Bai et. al., 2015)

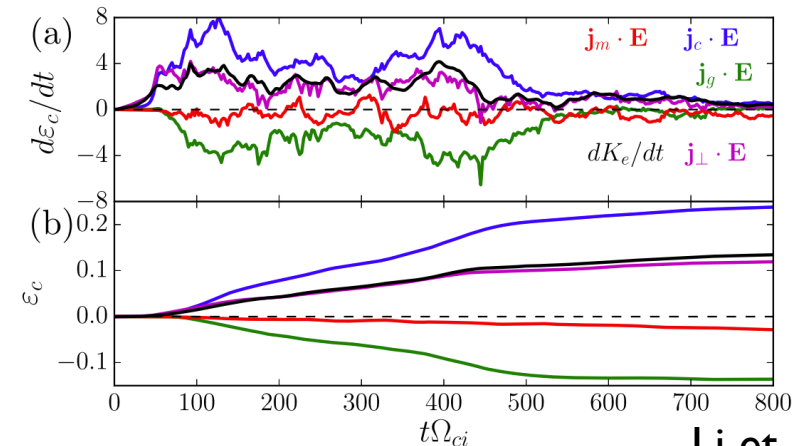


Particle acceleration in reconnection

- Particle acceleration during nonrelativistic reconnection
 - The **Fermi mechanism** has been proposed and demonstrated to be the dominant process for high-energy particle acceleration during reconnection. (Drake et al., 2006)
 - While injection requires kinetic physics, particle acceleration in reconnection can be largely understood from the guiding center approximation.



Ji et. al., 2022

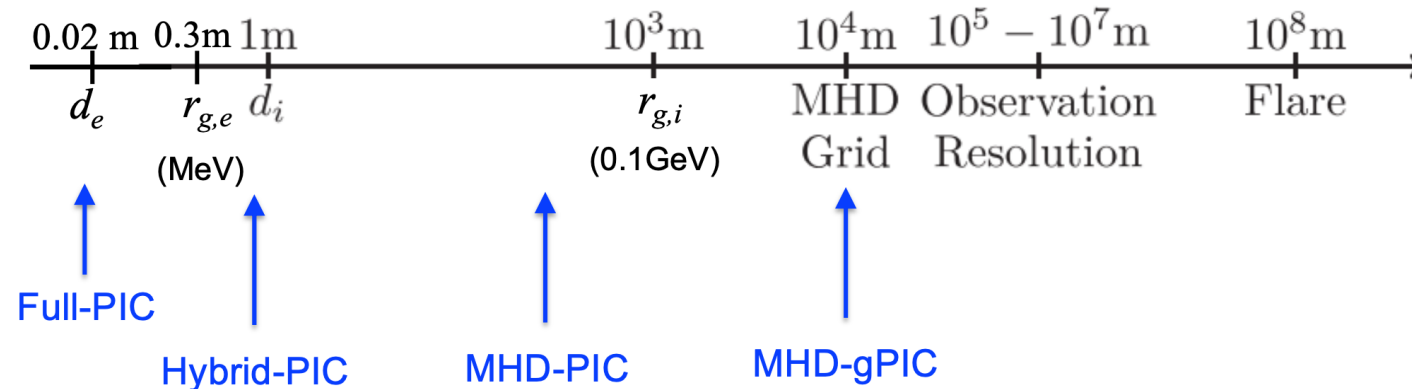


Li et. al., 2015

Issue with scale separation

- Dramatic scale separation involved in solar reconnection problem with particle acceleration.
- MHD-PIC: treat background plasma by MHD, while non-thermal particles are kinetic (Bai et. al., 2015)
- Represent particle by its guiding center (e.g. *kglobal*, Drake et. al., 2019; Arnold et. al., 2019; Yin, et. al., 2024)

➡ MHD-guiding-center-PIC model (MHD-gPIC model)



Basic formulation: particles

- Gyro-radii of particles \ll MHD grid size

$$\frac{d\mathbf{p}_k}{dt} = q\mathbf{E} + q\frac{\mathbf{v}_k}{c} \times \mathbf{B}$$

phase-averaging $\Delta t = \min\left(\Delta t_{\text{MHD}}, \frac{\Delta x}{v_k}, \frac{r_g}{v_{g\perp}}\right)$

$$\mathbf{v}_k = v_{\parallel} \mathbf{b} + \mathbf{v}_{g\perp} + \mathbf{v}_{\kappa} + \mathbf{v}_{\nabla B} + \mathbf{v}_{\text{speiser}}$$

curvature drift
Grad B drift

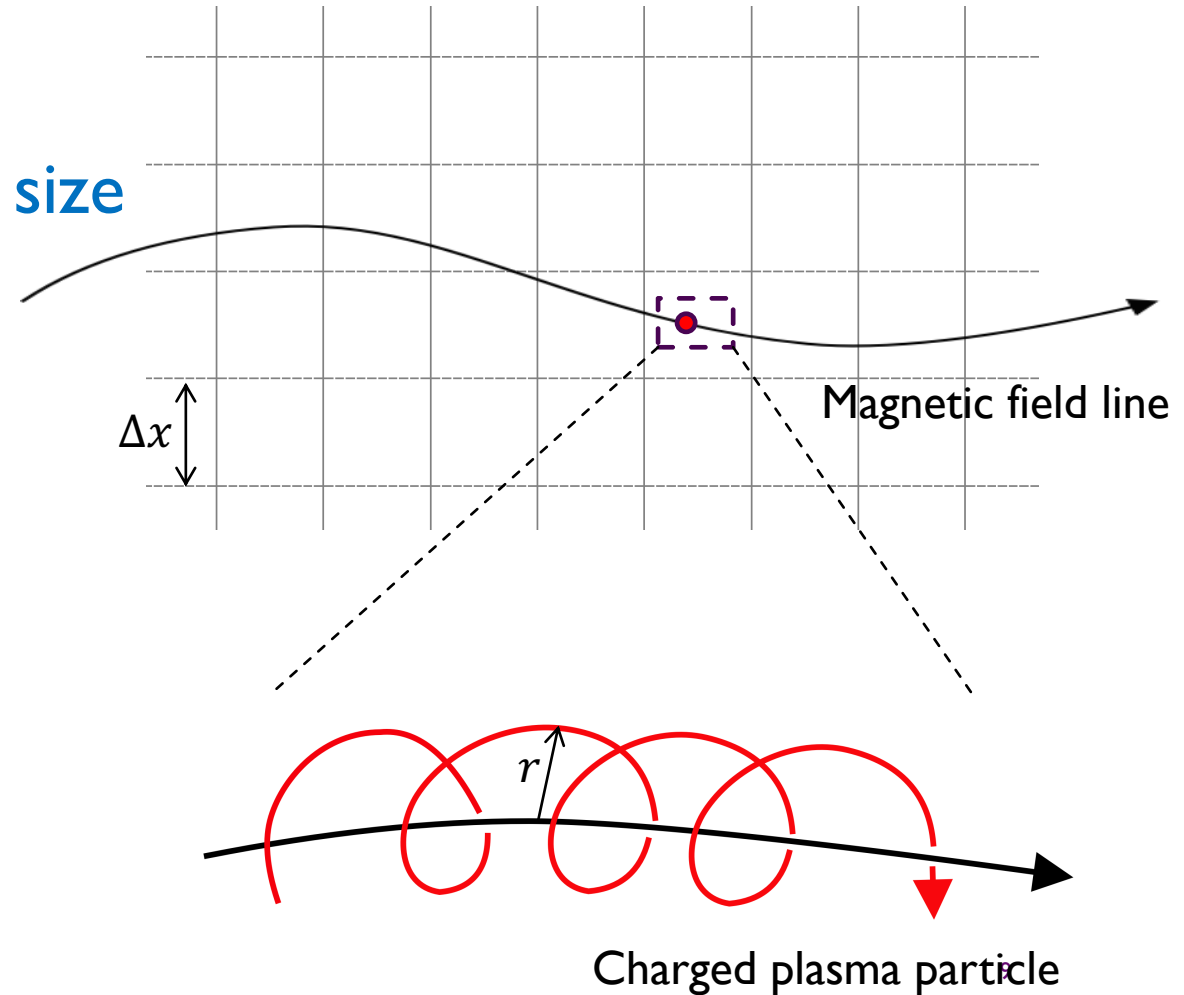
$$\mu_k \equiv \frac{p_{k\perp}^2}{2m_k B} = \text{const}$$

$E \times B$ drift

$$\frac{d}{dt} p_{k\parallel} = (p_{k\parallel} - \gamma_k v_{g\parallel}) \mathbf{v}_g \cdot \boldsymbol{\kappa} - \frac{\mu_k}{\gamma_k} \mathbf{b} \cdot \nabla B - eE_{\parallel}$$

change of b direction
mirror effect

Northrop, 1963

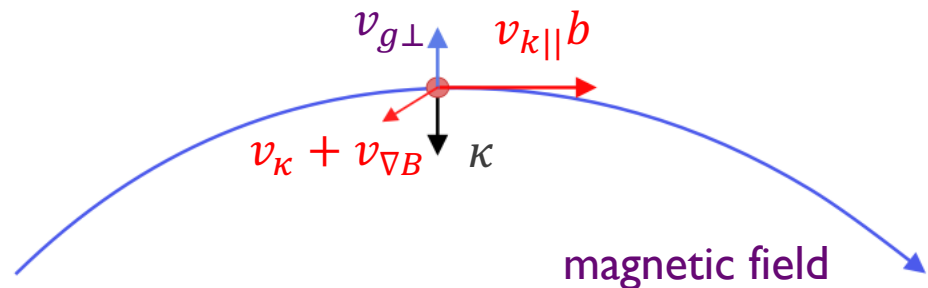


Implementing MHD-gPIC formalism in Athena++

■ Particles

$$\mathbf{v}_k = v_{\parallel} \mathbf{b} + \mathbf{v}_{g\perp} + \mathbf{v}_{\kappa} + \mathbf{v}_{\nabla B} + \mathbf{v}_{\text{speiser}}$$

$$\mathbf{x}_{t+\Delta t} = \mathbf{x}_t + \mathbf{v}_k \Delta t + \frac{1}{2} v_{\parallel}^2 \kappa \Delta t^2$$



$$\frac{d}{dt} p_{k\parallel} = (p_{k\parallel} - \gamma_k v_{g\parallel}) \mathbf{v}_g \cdot \boldsymbol{\kappa} - \frac{\mu_k}{\gamma_k} \mathbf{b} \cdot \nabla B - e E_{\parallel}$$

$$\begin{aligned} & \vec{v}_g, \\ & \nabla_{\parallel} \vec{v}_g \\ & \vec{B}, \\ & \vec{\kappa} + \nabla_{\parallel} B \vec{b}, \\ & E_{\parallel} \end{aligned}$$

$$\begin{aligned} & T_{p\parallel}, \\ & P_{p\perp}, \\ & p_{p\parallel}, \\ & n_p \end{aligned}$$

■ MHD

Source terms:

Momentum:

$$\mathbf{F}_{\text{gPIC}} \equiv -R (\nabla P - \mathbf{J} \times \mathbf{B})_{\perp} + (1 - R) \mathbf{F}_{p\perp} + \mathbf{F}_{p\parallel},$$

$$\mathbf{F}_{p\perp} \equiv \Delta T_p \boldsymbol{\kappa} + 2\rho_{\text{pi}} \Delta u_{\text{pi}\parallel} \frac{D\mathbf{b}}{Dt},$$

$$\mathbf{F}_{p\parallel} \equiv [q_{\text{pi}} E_{\parallel} - \Delta T_{\text{pe}} \nabla_{\parallel} \ln B + \nabla_{\parallel} (T_{\text{pe}\parallel} - P_{p\perp})] \mathbf{b}.$$

Energy:

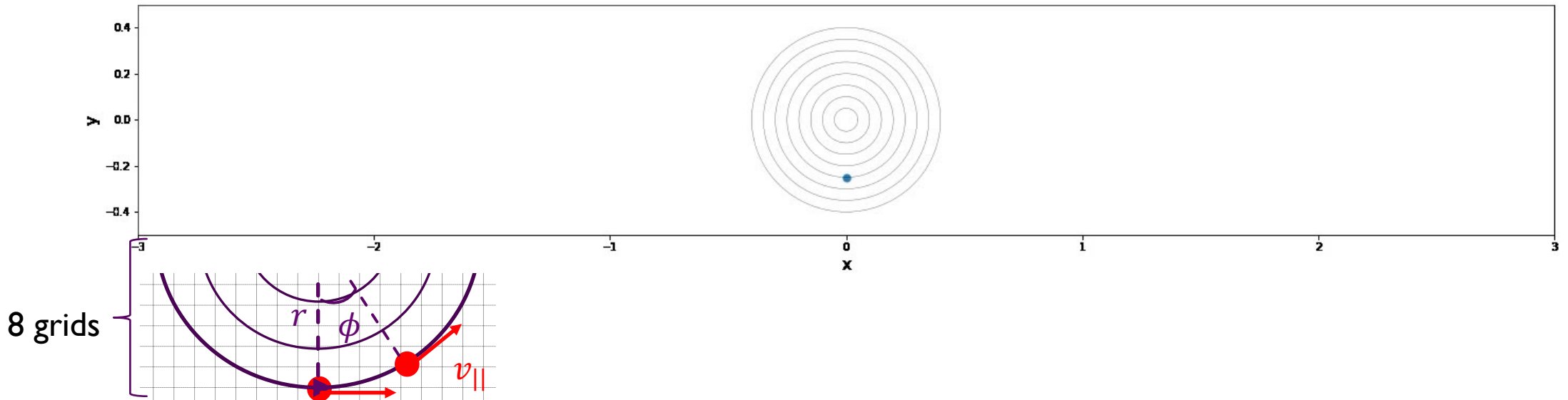
$$W_{\text{gPIC}} = \mathbf{F}_{\text{gPIC}} \cdot \mathbf{u}_{\perp} + J_{p\parallel} E_{\parallel} - P_{p\perp} \nabla \cdot \mathbf{u}_{\perp} + \nabla \cdot (\mathcal{E}_{\text{fe}} \Delta \mathbf{u}_{\text{fe}} - P_{p\perp} \mathbf{u}_{\parallel} \mathbf{b}),$$

Electric field:

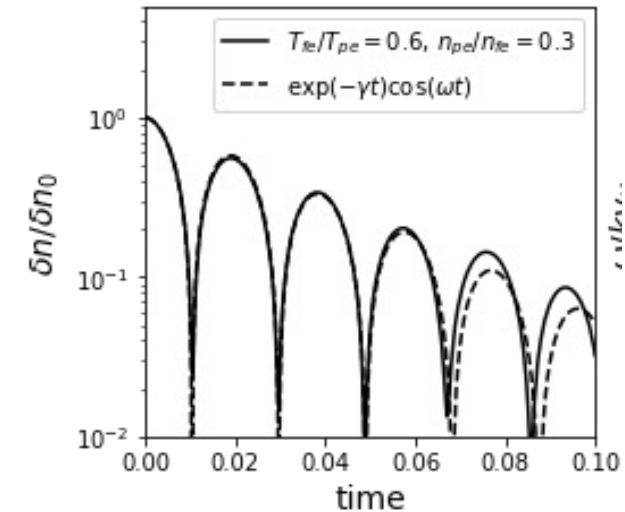
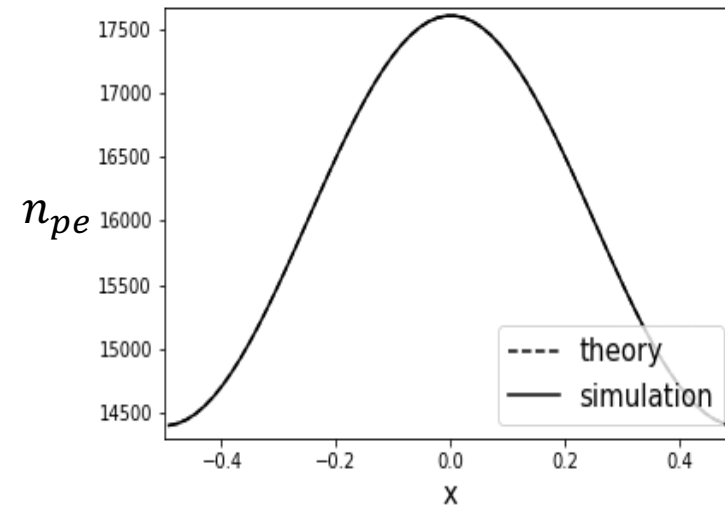
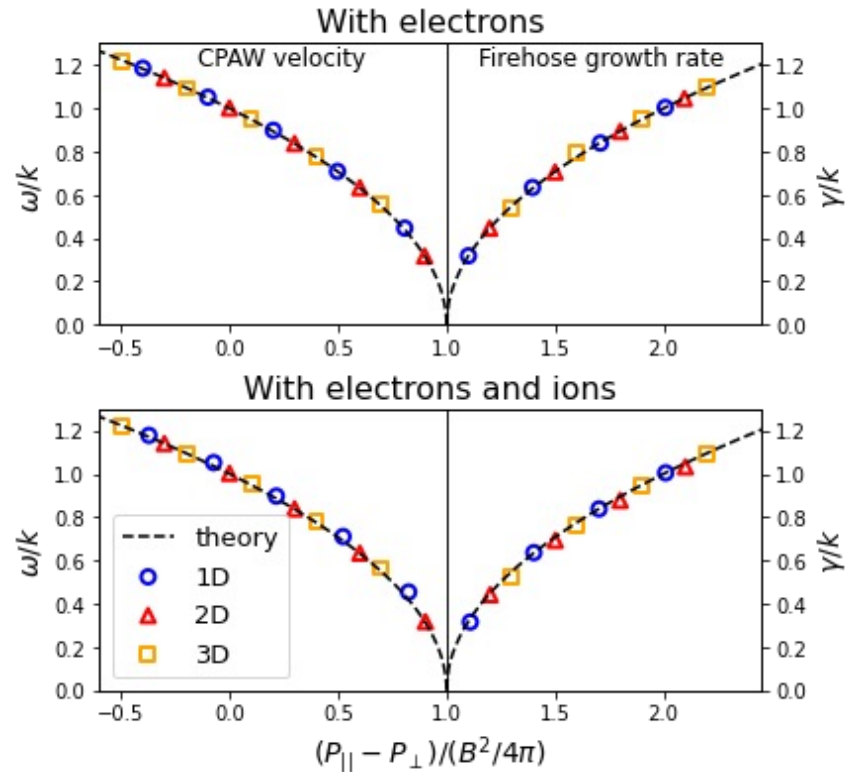
$$E_{\parallel} = \frac{1}{n_e e} \mathbf{b} \cdot \nabla \cdot \mathbf{P}_e$$

Benchmark test: particle motion

- Particle motion along a moving 2D field loop
- The particle can maintain a stable position on the same magnetic field line over multiple periods and at a constant velocity in the moving reference frame.



Benchmark test: firehose instability & Landau damping

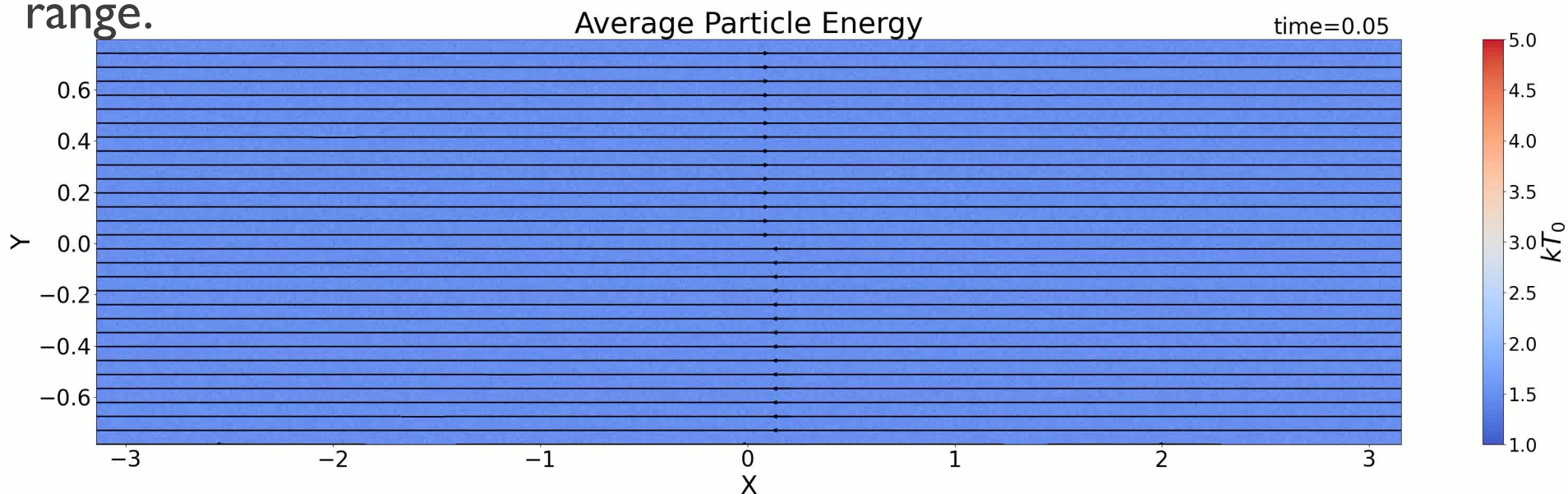
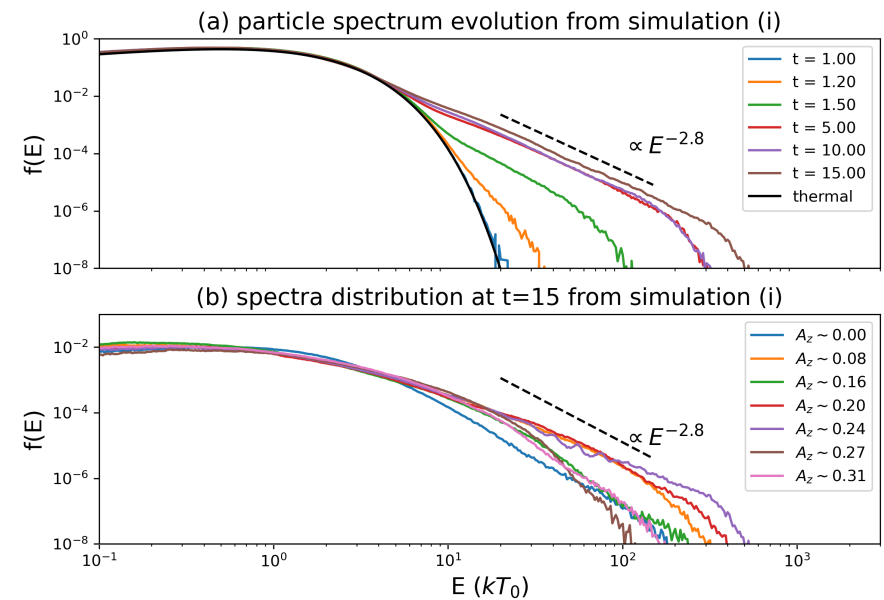


Landau damping of electron acoustic waves

With anisotropic particle pressure: firehose instability

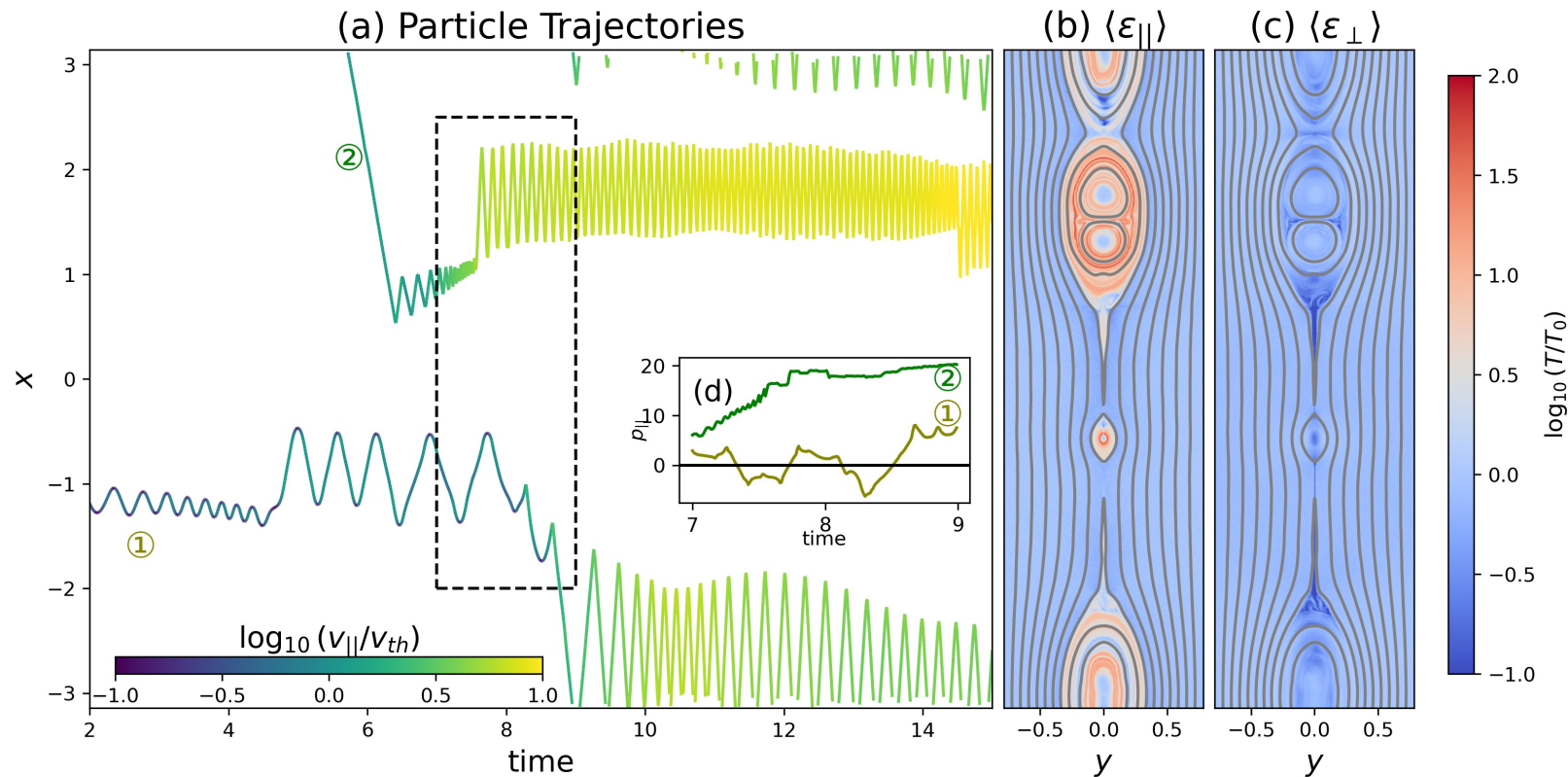
Test: 2D reconnection

- 2D non-relativistic reconnection,
 - Force-free magnetic field, $\frac{B_g}{B_0} = 0.1, \beta = 0.25,$
 - Mesh size: 2048×512 , periodic boundary in X direction, reflecting in Y direction
- Part of particles are accelerated and form power-law spectrum in a specific energy range.



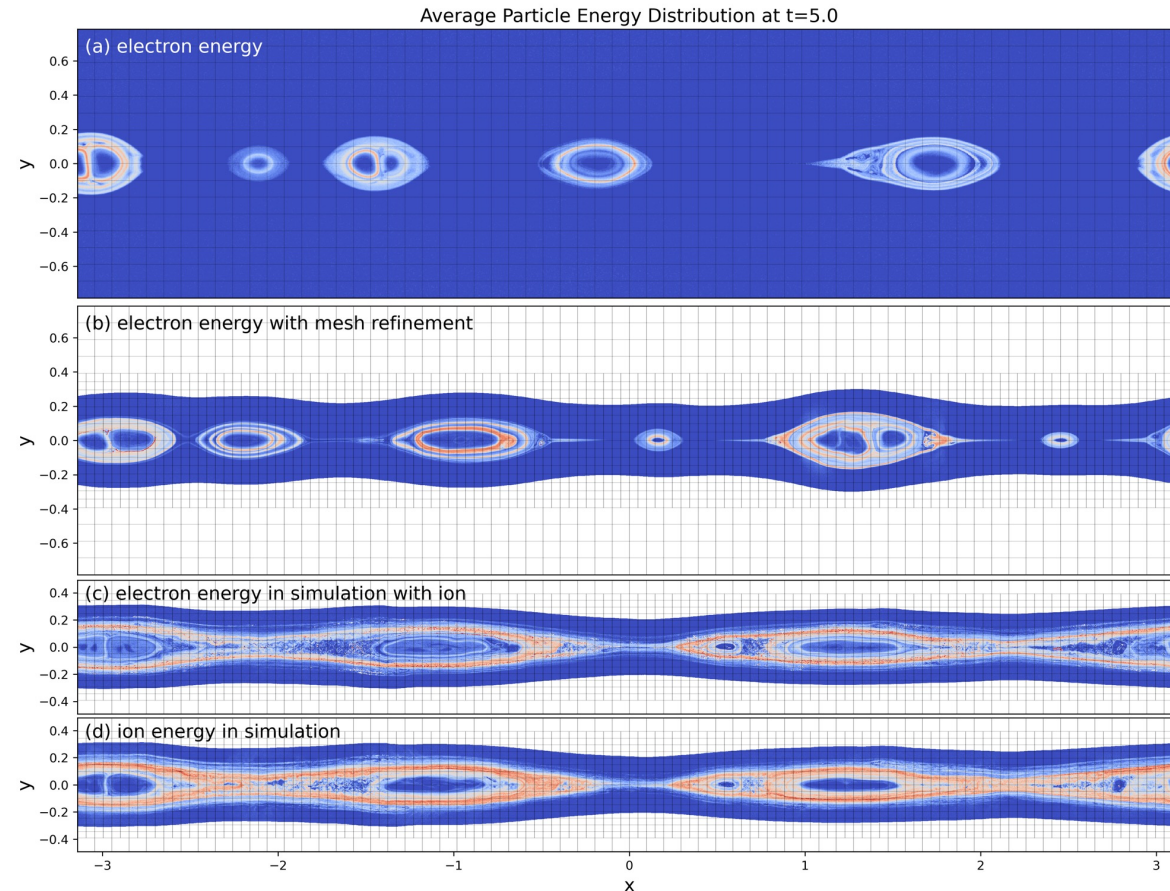
Test: 2D reconnection

- Particles are accelerated during the shrinking and merging of plasmoids
- Due to the confinement, high-energy particles cannot access the regions where the acceleration is most active.



Test: 2D reconnection with mesh refinement

- A higher-resolution test, conducted by applying mesh refinement, confirms the above conclusions.
- Ions are accelerated more efficiently as their initial velocity is much smaller than the shrink velocity of plasmoids.
- Feedback of ions may be able to make reconnection faster as they take away more energy near the X-points.



Toward Global Multi-Scale Simulation

- Challenges:

- Unstable for low β simulation

← combine with dual energy formalism

- 3D simulations: how to use fewer particles without missing important physics?

← particle injection recipe from small scale PIC simulations + mesh refinement

- The guiding center approximation is not always valid

← subgrid physics (e.g. pitch angle diffusion, spatial diffusion)

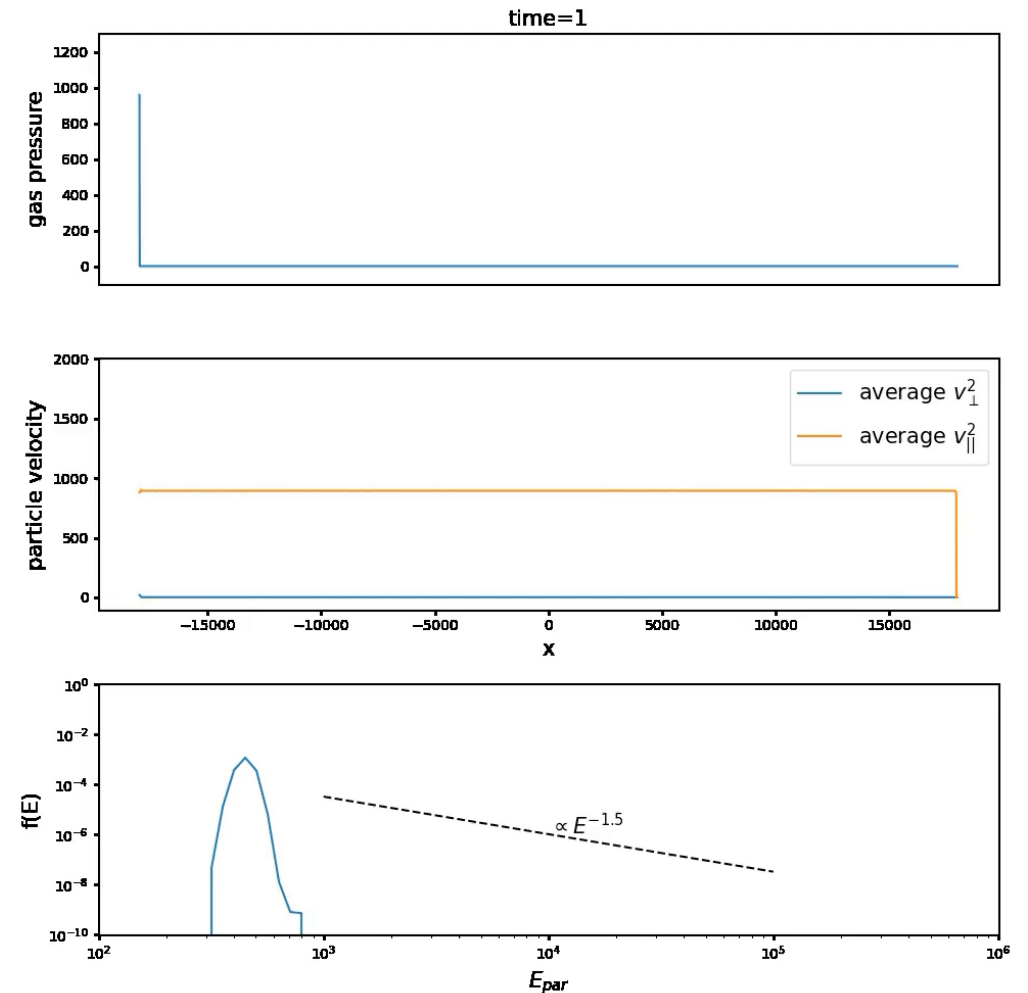
Addition of pitch angle diffusion

- Turbulence-based pitch-angle diffusion as sub-grid model

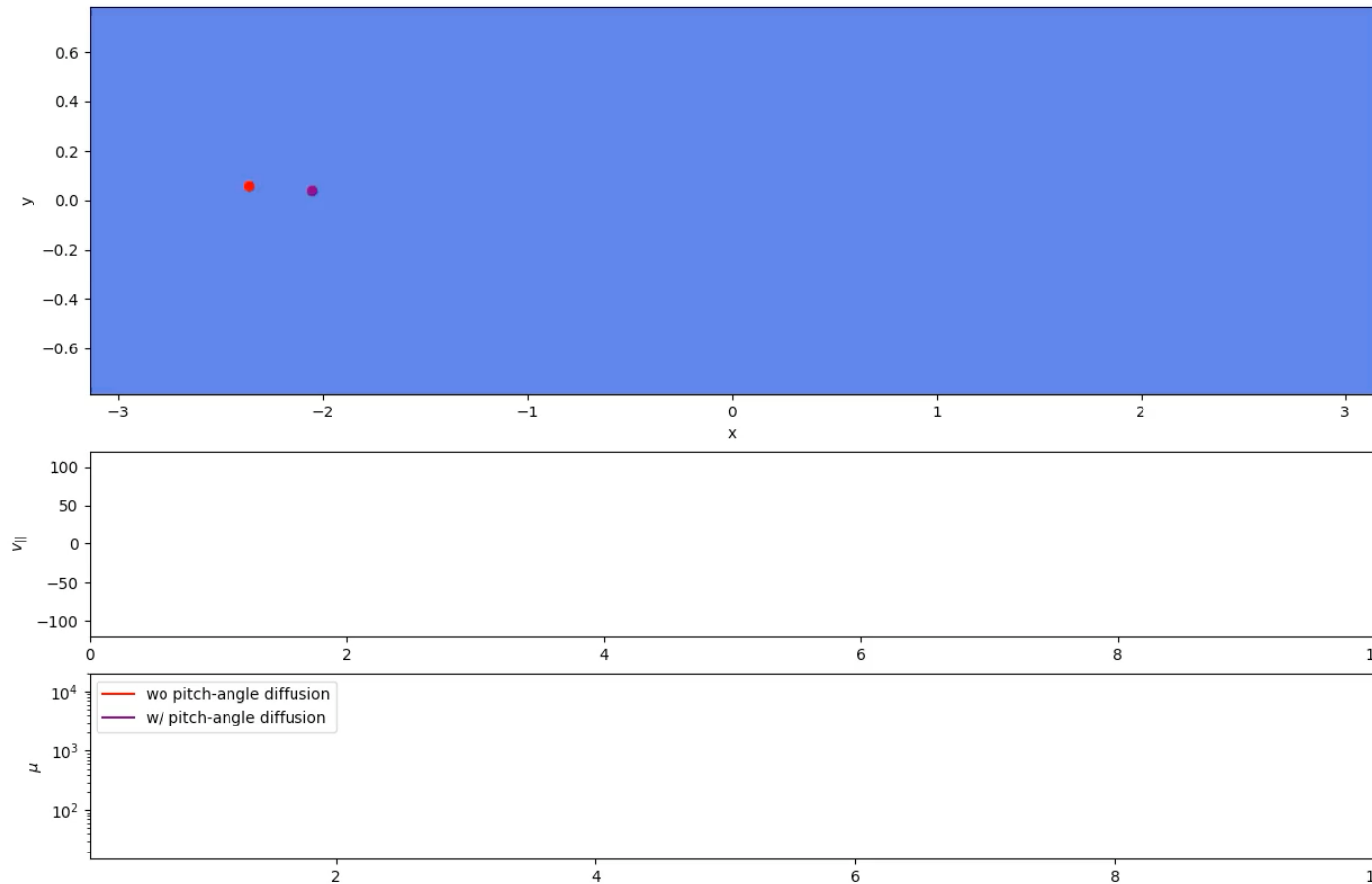
$$D_{\mu\mu}(v, B_0, \mu) = \frac{\pi}{4} \frac{\Omega_0^2}{B_0^2 v} \frac{1 - \mu^2}{|\mu|} P \left(k_{\parallel} = \frac{\Omega_0}{\mu v} \right)$$

- Enables simulation of particle acceleration in shocks

=> comprehensive modeling of particle acceleration in solar flares.



Particle acceleration in reconnection with $D_{\mu\mu}$



- Convert parallel to perpendicular energy via pitch-angle scattering, amplifying the magnetic moment.
- Trap particles effectively within magnetic bottles.
- Achieve multiple accelerations of a single particle within one plasmoid. (Chen et al. 2024)

Summary

- **Issue with scale separation for non-relativistic reconnection**
 - Conventional PIC methods, even with MHD-PIC method, suffer major difficulties.
 - Particle acceleration in reconnection involves little gyro-scale physics.
- **Development of MHD-guiding-center-PIC (MHD-gPIC) code**
 - Built upon the MHD-PIC module in Athena++.
 - Carefully formulated and implemented: Galilean invariant, low noise, accurate feedback.
- **Test simulations of non-relativistic reconnection**
 - Fermi acceleration during islands contraction/merger.
 - Power law spectrum in test particle simulations, more efficient but more complex behavior with particle feedback.
- **Future prospects**
 - More sub-grid physics (injection/particle diffusion), toward global systems.