PRELIMINARY

10th East Asia Numerical Astrophysics Meeting

Investigating AMR-driven Discontinuities in Cosmological Hydrodynamical Simulation

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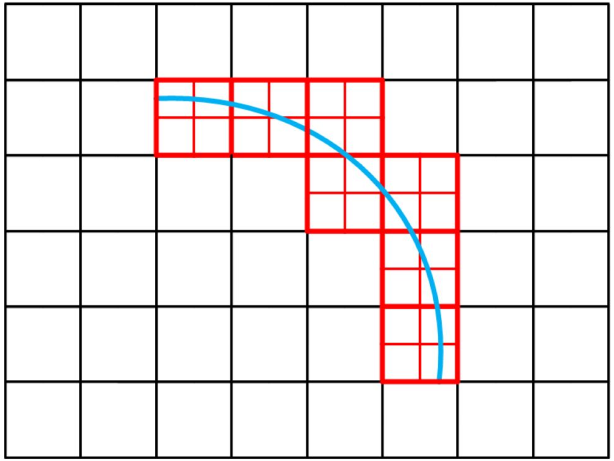
Background Image: a snapshot from Horizon Run 5

Adaptive Mesh Refinement (AMR) Method

- In the **uniform grid method**, grid cells have **a fixed spatial resolution**.
- The **AMR method** refines grid cells to smaller ones **to get higher resolution**.
- The **refinement conditions** can be various; high density, steep gradients, pre-selected region, etc.



- **Typical refinement condition:** $m_{cell} \geq m_{crit}$
  (in cosmological simulations)

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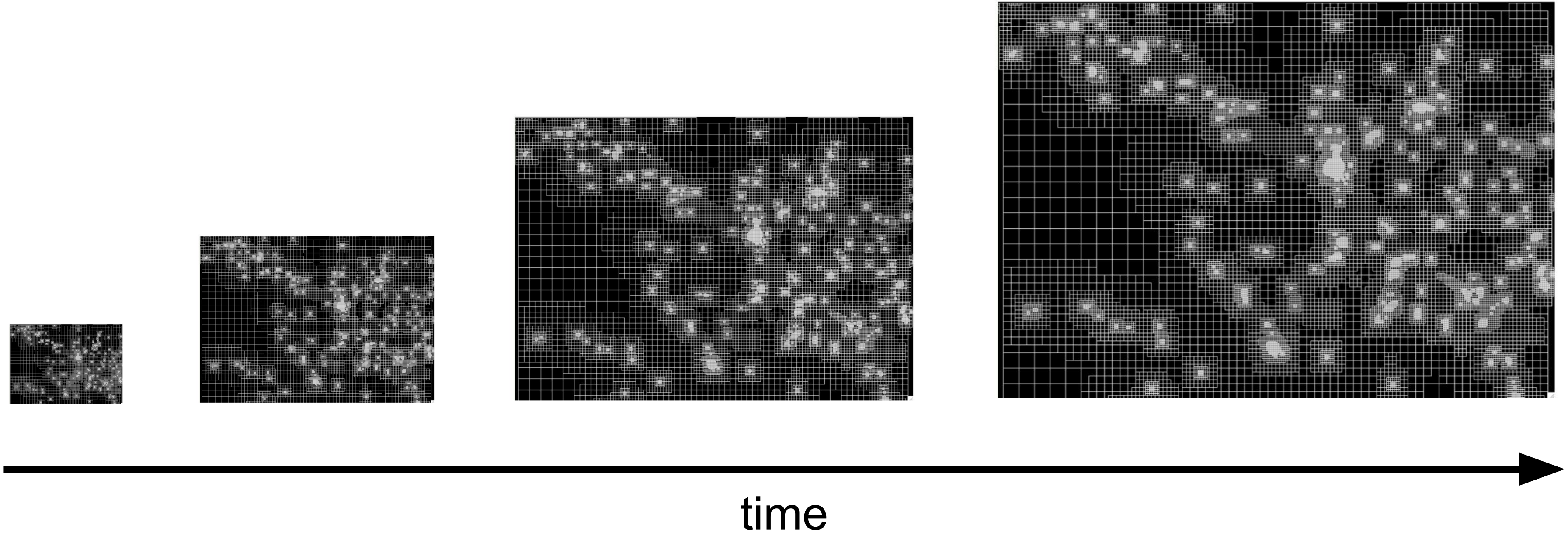
In the Cosmological Context ···

the physical size of the cells scales up with cosmic expansion
= effective resolution continually degrades



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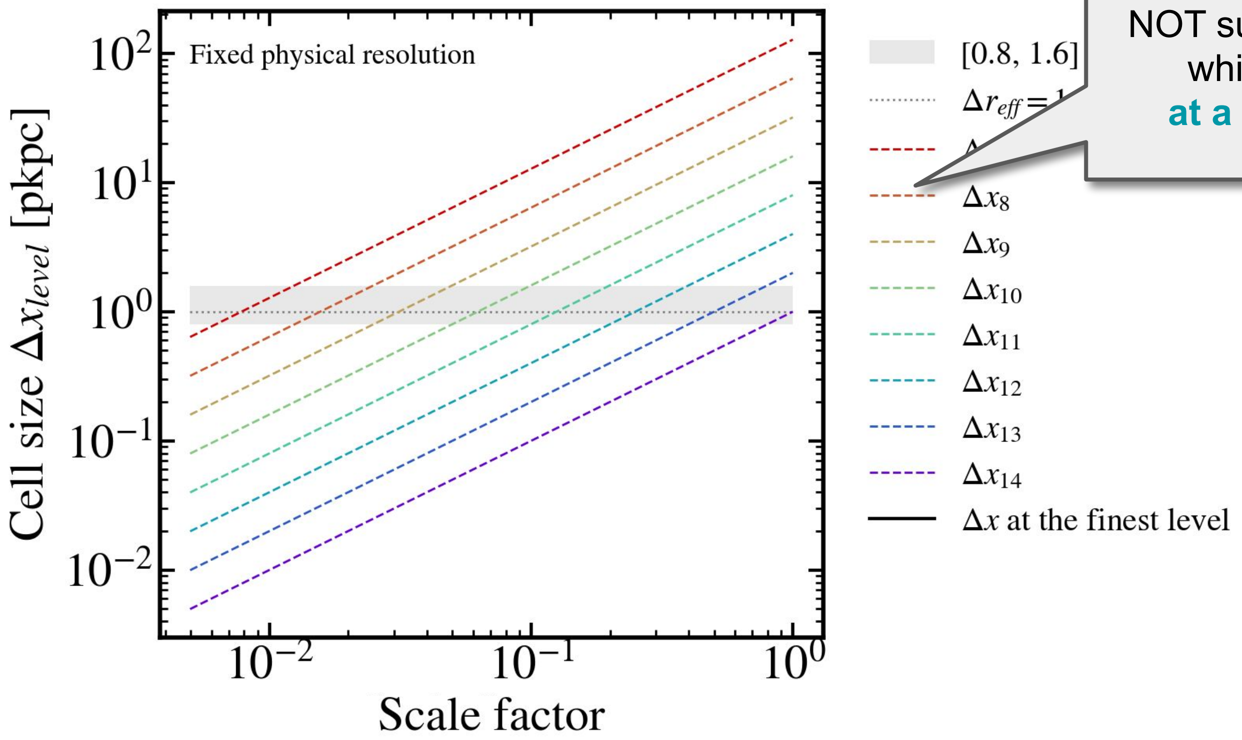
Grid Holdback Method

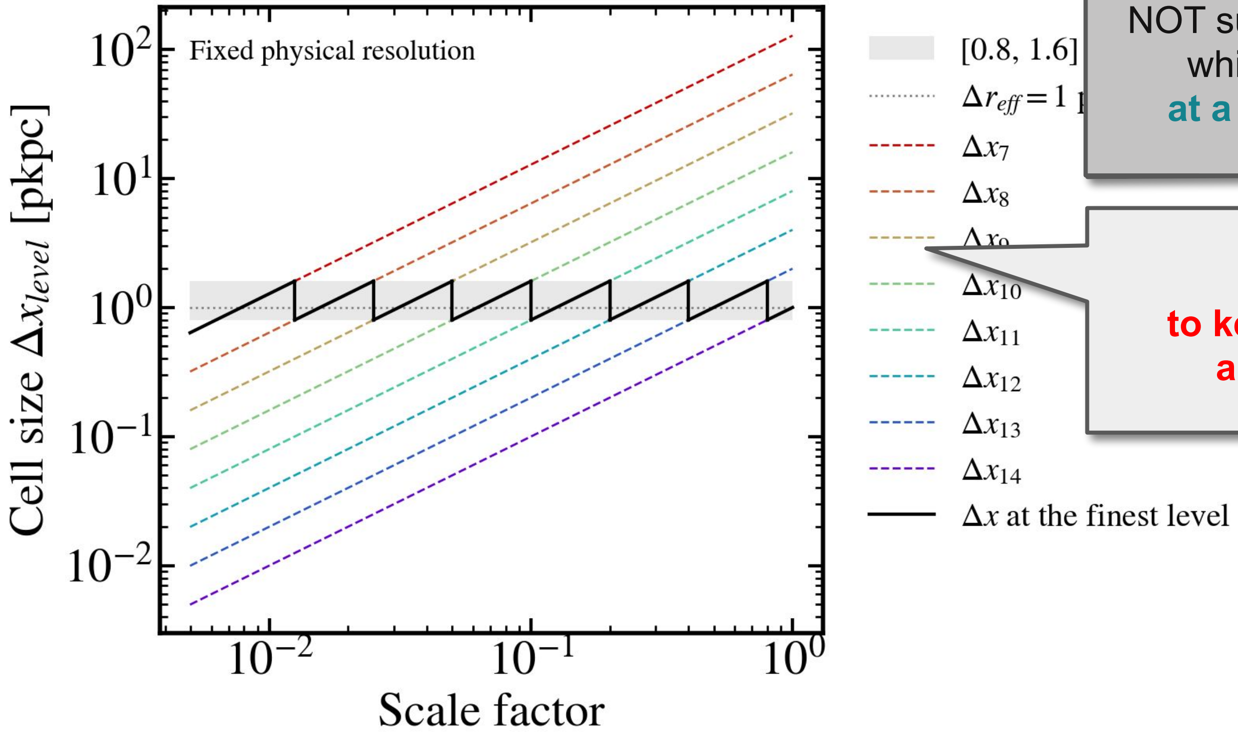
- refinement condition: $m_{\rm cell} > m_{\rm crit}$



NOT suitable with **subgrid models**, which is designed to operate **at a specific, fixed resolution**

Grid Holdback Method

- refinement condition: $m_{\rm cell} > m_{\rm crit}$



NOT suitable with **subgrid models**, which is designed to operate **at a specific, fixed resolution**

Holdback method
to keep the spatial resolution approximately constant

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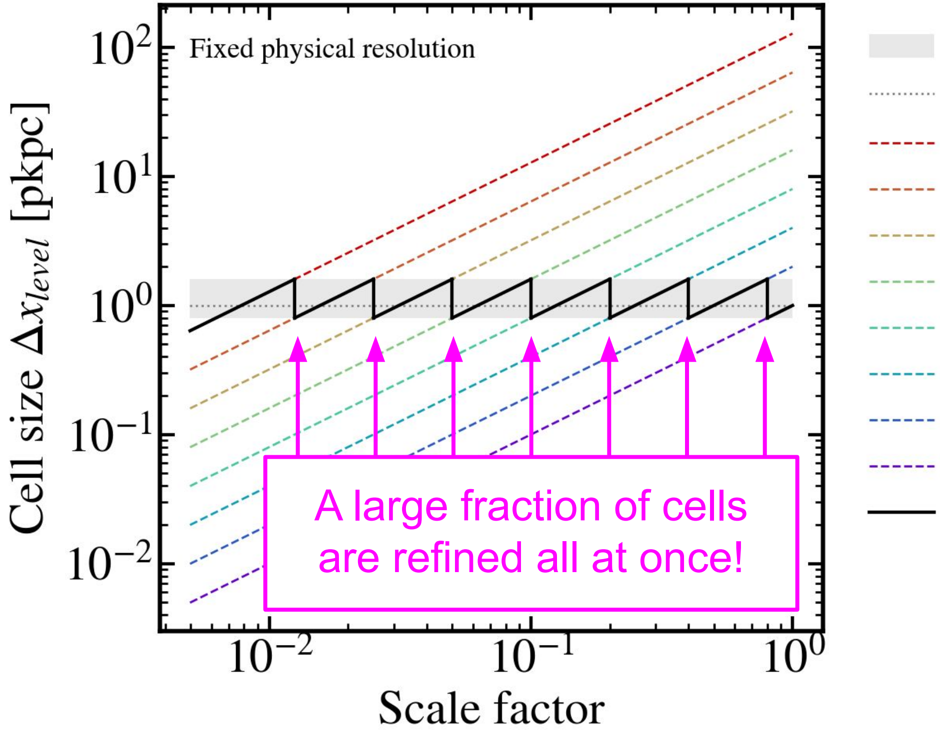
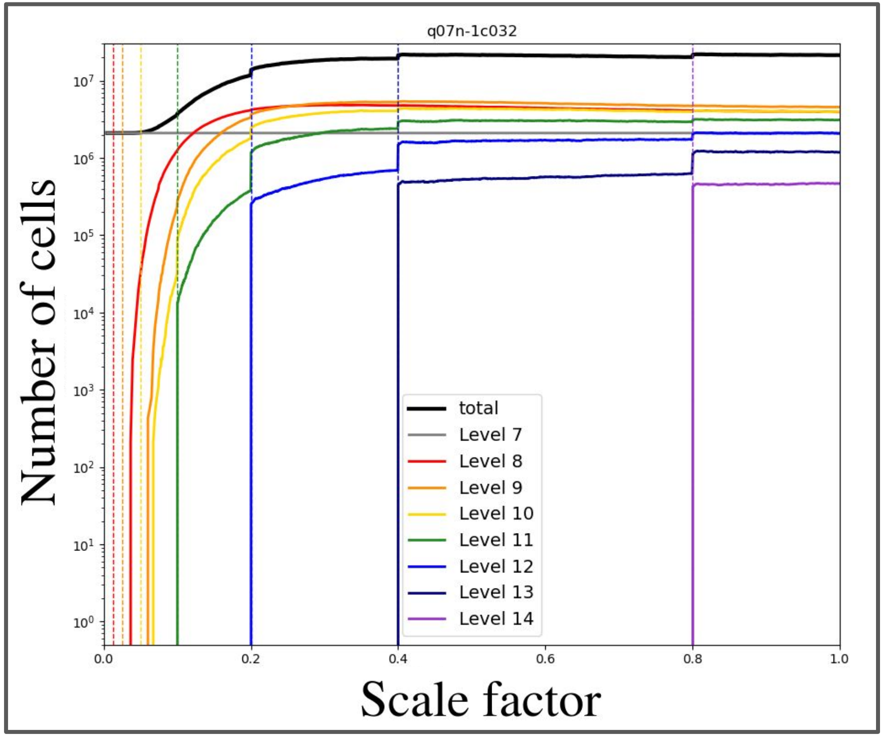
Grid Holdback Method

- refinement condition: $m_{\text{cell}} > m_{\text{crit}}$





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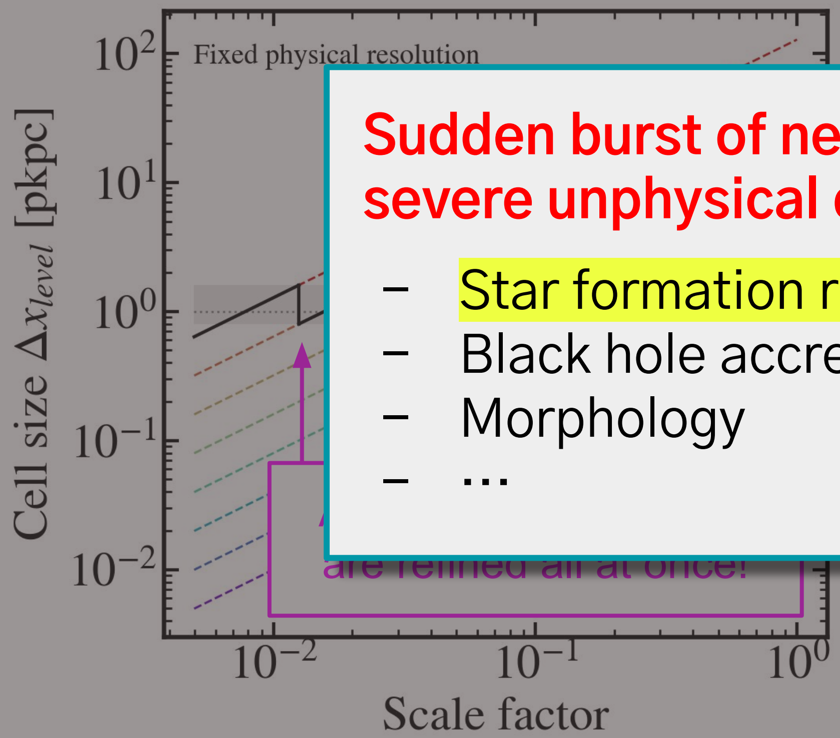
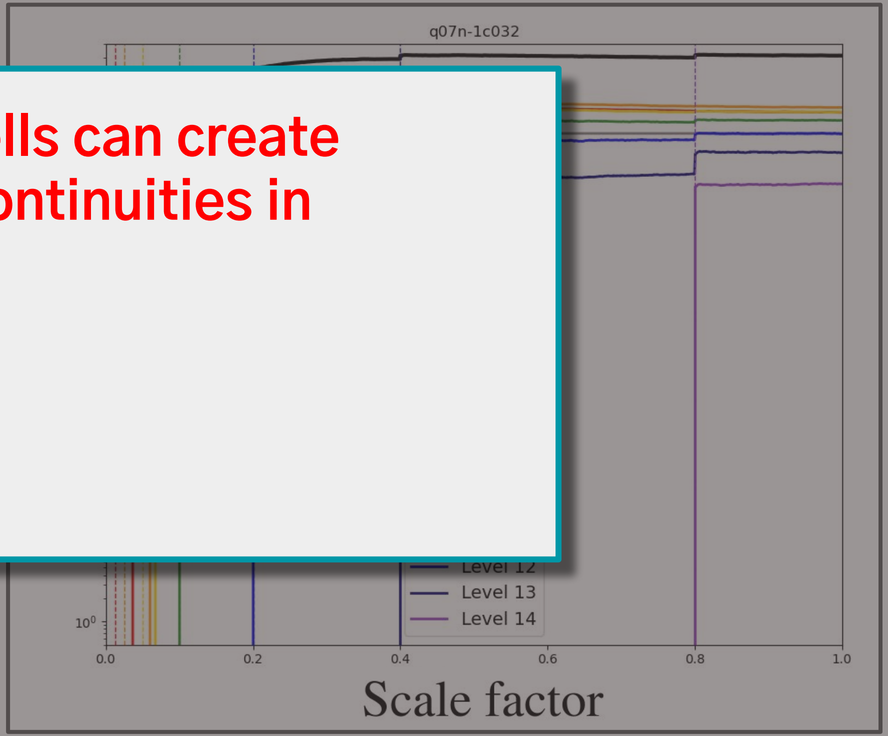
Grid Holdback Method

- refinement condition: $m_{\rm cell} > m_{\rm crit}$





Sudden burst of new cells can create severe unphysical discontinuities in

- Star formation rate
- Black hole accretion
- Morphology
- …

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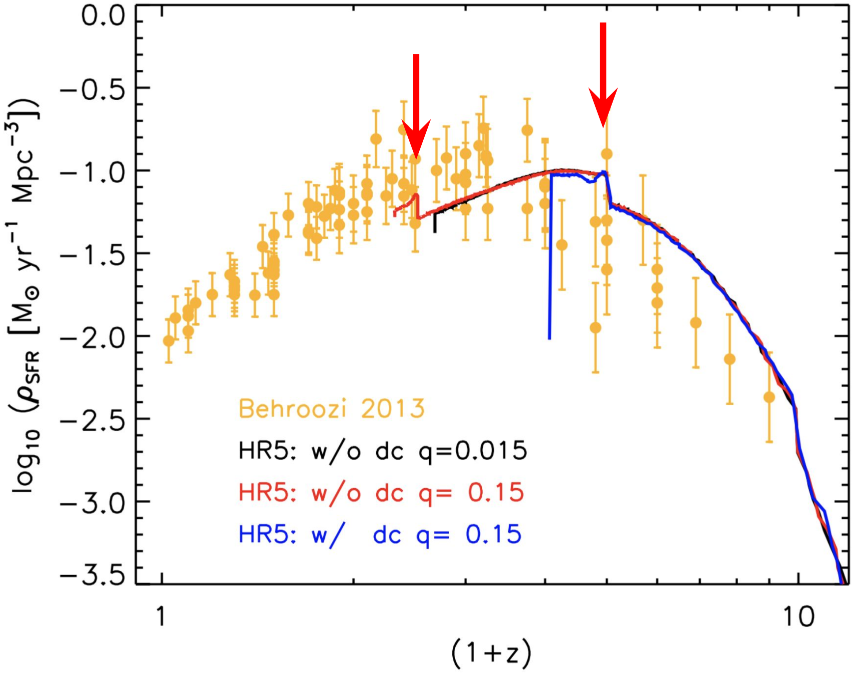
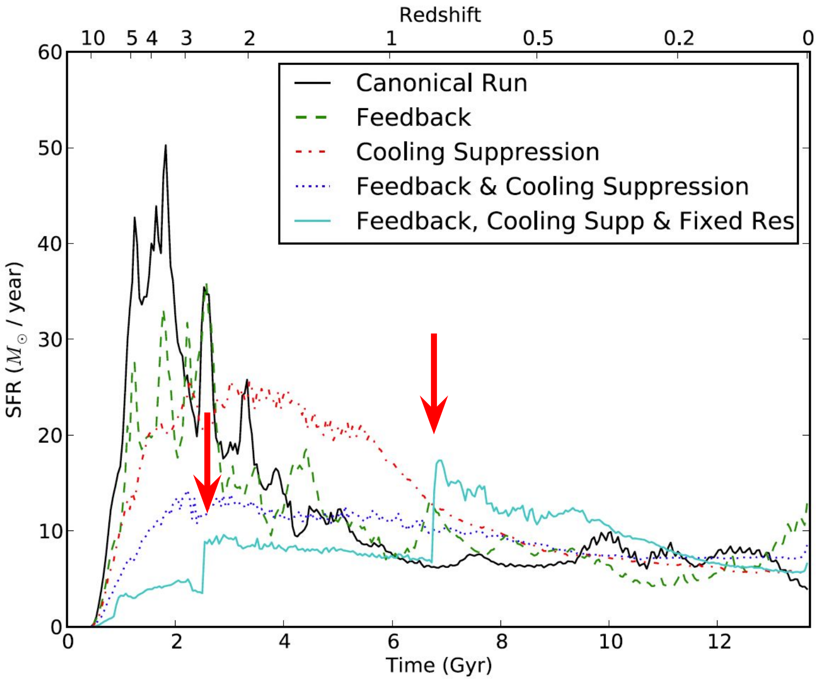
Unphysical Jumps in Star Formation History

- **Global jumps** in cosmic SFR density
  (e.g.) **J. Lee, J. Shin et al. 2021** (Horizon-Run 5; *RAMSES*)



- **Local SFR jumps** even in the individual galaxy
  (e.g.) **Hummels & Bryan 2012** (*ENZO*)



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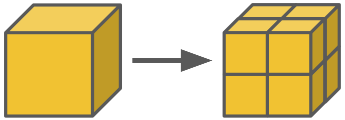
What Causes the Jumps?

Two key factors behind the SFR jumps

1) Refinement Strategy

- **Mass-based refinement:** Quasi-Lagrangian scheme



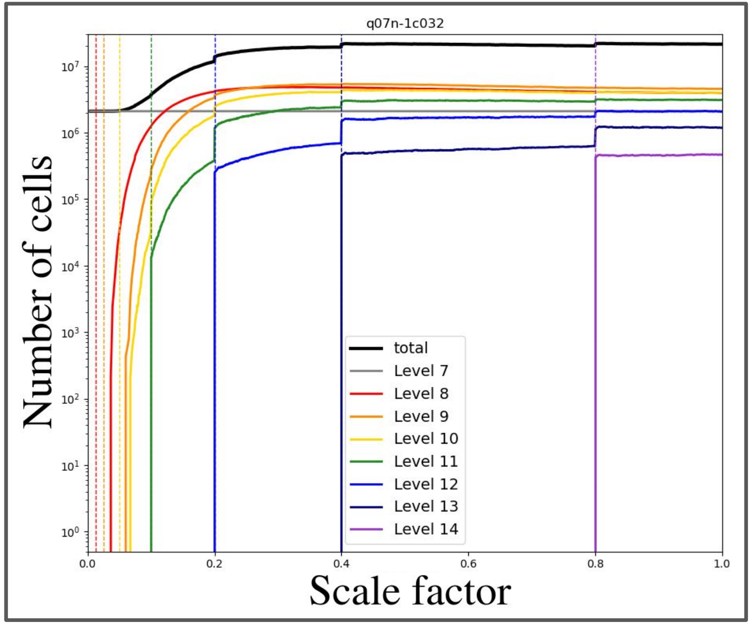
$$m_{\mathrm{cell}} > m_{\mathrm{crit}} = n \times m_{\mathrm{sph}}$$

where n = number of child cells = 8
m_{sph} = initial average gas cell mass

- **Grid holdback method**:
  - New AMR levels only allowed at specific times
  - Many cells are refined all at once

2) Star Formation Recipe

- **SF condition:** $n_H > 0.1 cm^{-3}$
- **Only leaf cells** (which have no child cells)



After new-level refinement:

Many small, dense leaf cells are created
Easier to trigger star formation

⇒ Numerical starbursts (jumps in SFR)

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Our solution

- **Set a target resolution (Δr) to match with subgrid physics model**
- **No holdback method, but modified:**

1. **Refinement method**:
   - **Harder for small cells to refine**
   - Not arbitrary → physically motivated

   most crucial part!

2. **Gravity solver**: *[ongoing]*
   - Use a **smooth density field** in Poisson equation
3. **Hydro solver**:
   - Apply **temperature floor** (Truelove et al. 1997) to prevent artificial fragmentation
   - Resolve Jeans length ≥ 4 x target resolution ($\lambda_J \geq 4\ \Delta r$)

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Our solution – 1. Modified Refinement Strategy

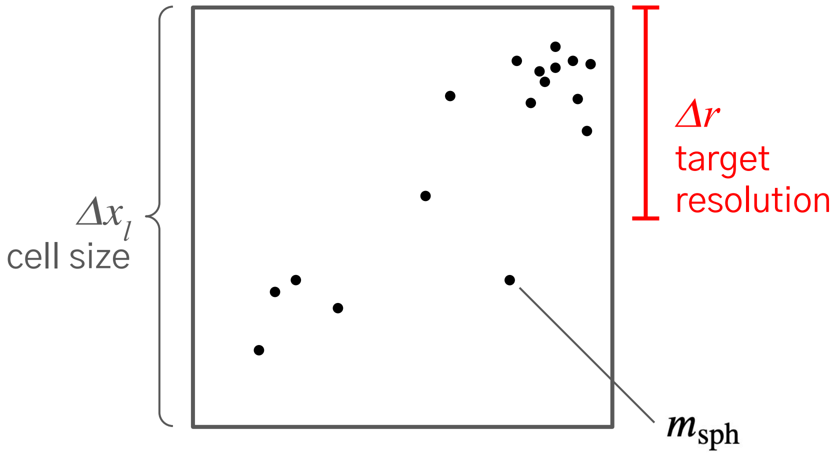
Refinement condition:

if $n = 8$

$$m_{\mathrm{cell}} > m_{\mathrm{crit}} = n \times m_{\mathrm{sph}}$$

quasi–Lagrangian:

Refinement goes beyond the target resolution



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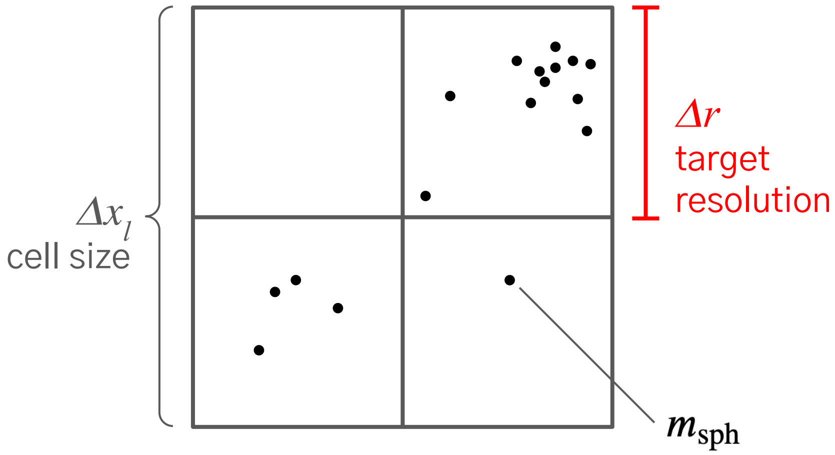

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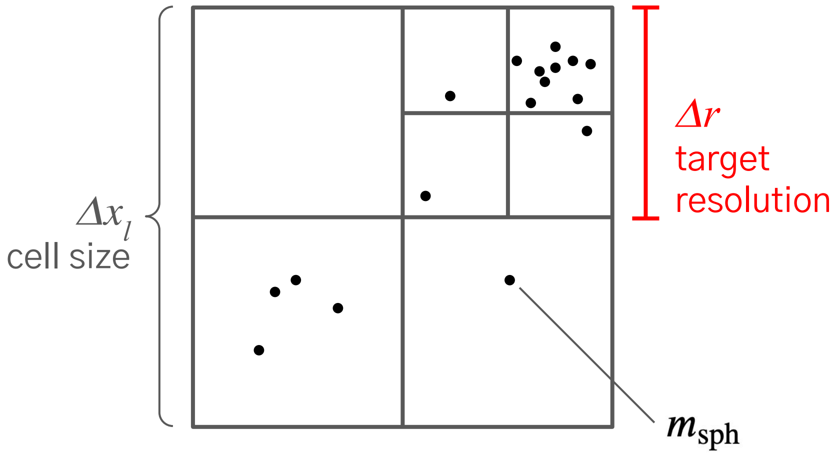

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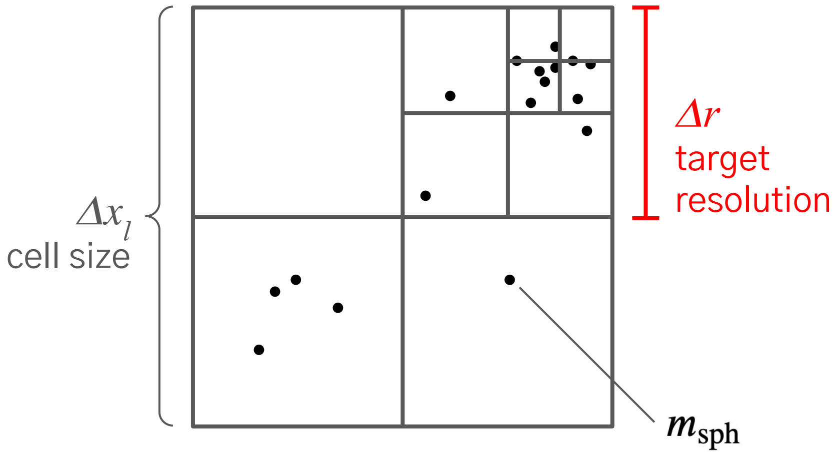

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Our solution – 1. Modified Refinement Strategy

Refinement condition:

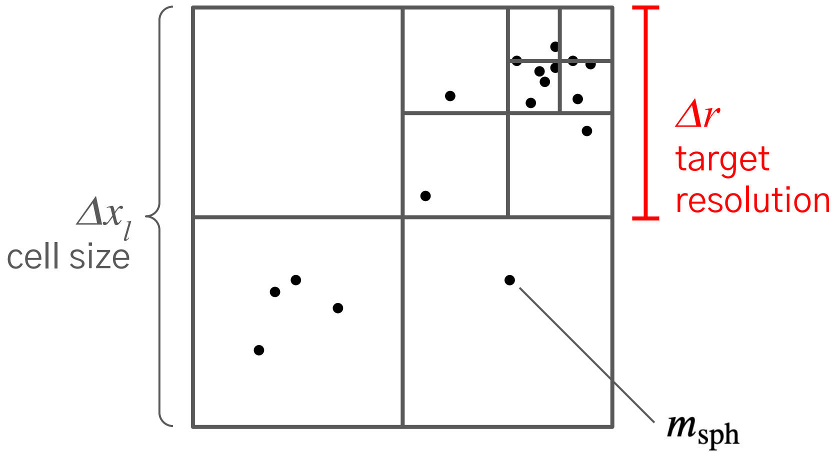
$$m_{\rm cell} > m_{\rm crit} = n \times m_{\rm sph} \times \left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)} \right)^3$$

if $n = 8$

$\left\{ \begin{array}{ll} = 1;\ \text{quasi-Lagrangian} & \text{if } \Delta x \geq 2\,\Delta r \\ > 1;\ \text{sub-Lagrangian} & \text{if } \Delta x < 2\,\Delta r \end{array} \right.$

quasi-Lagrangian:

Refinement goes beyond the target resolution



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Our solution – 1. Modified Refinement Strategy

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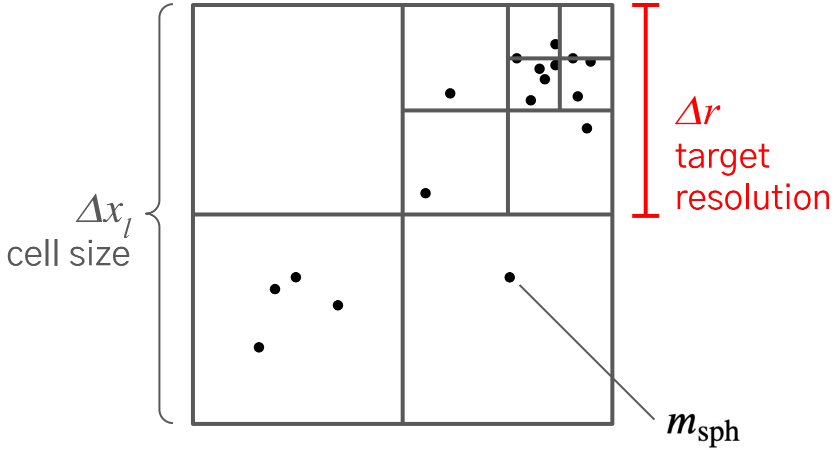
$$m_{\mathrm{cell}} > m_{\mathrm{crit}} = n \times m_{\mathrm{sph}} \times \left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)} \right)^3$$

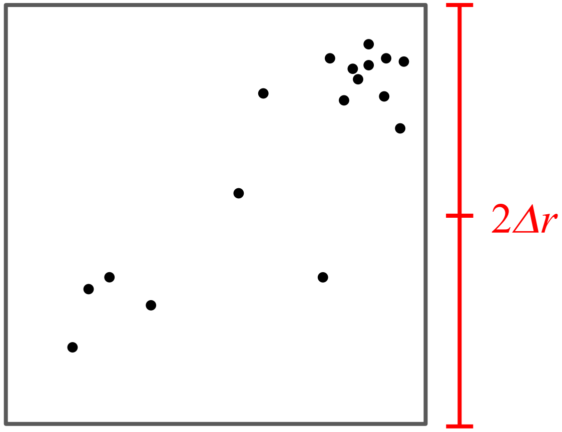
if $n = 8$

$\left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)}\right)^3$: = 1; quasi–Lagrangian if $\Delta x \geq 2\,\Delta r$; > 1; sub–Lagrangian if $\Delta x < 2\,\Delta r$

quasi–Lagrangian:
Refinement goes beyond the target resolution

sub–Lagrangian:
Suppress over–refinement at small scales





If $\Delta x_l = 2\Delta r$ then
$m_{\mathrm{crit},l} = 8 \times m_{\mathrm{sph}}$

For next level,
$\Delta x_{l+1} < 2\Delta r$
$m_{\mathrm{crit},l+1} = 64 \times m_{\mathrm{sph}}$

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Our solution – 1. Modified Refinement Strategy

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if $n = 8$

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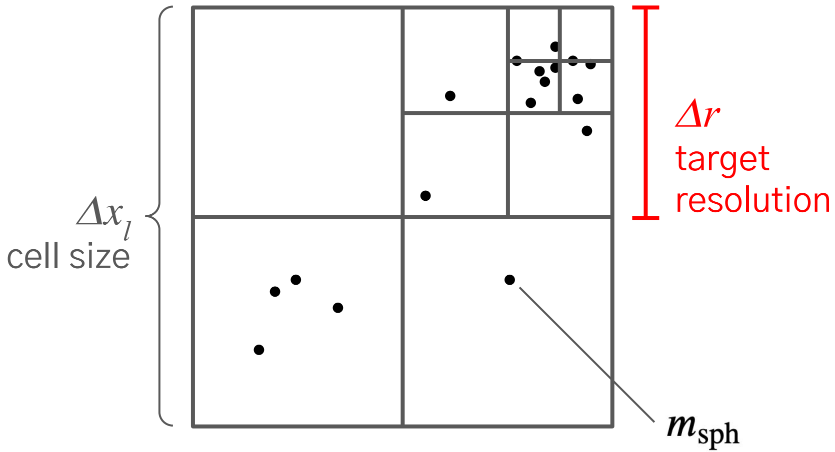
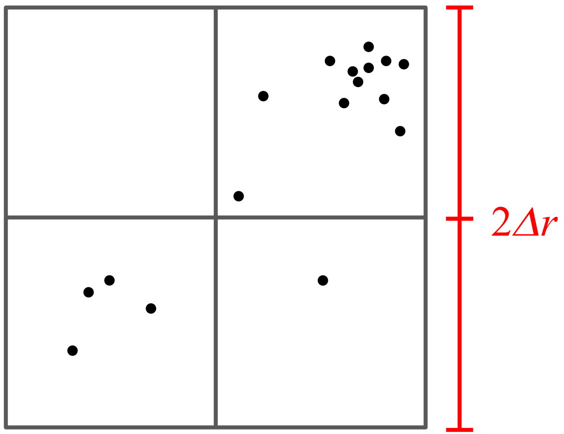
quasi–Lagrangian:
Refinement goes beyond the target resolution

sub–Lagrangian:
Suppress over–refinement at small scales





If $\Delta x_l = 2\Delta r$ then
$m_{{\rm crit},l} = 8 \times m_{\rm sph}$

For next level,
$\Delta x_{l+1} < 2\Delta r$
$m_{{\rm crit},l+1} = 64 \times m_{\rm sph}$

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Our solution – 1. Modified Refinement Strategy

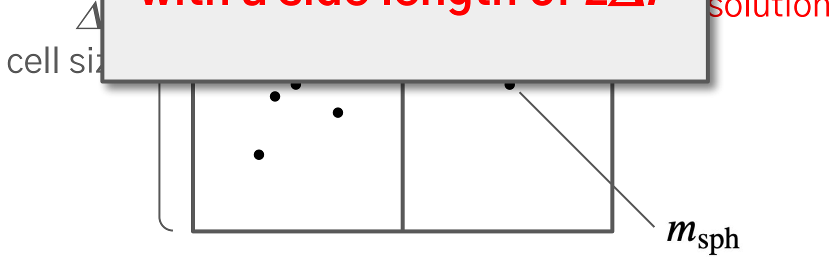
Refinement condition:

if $n = 8$

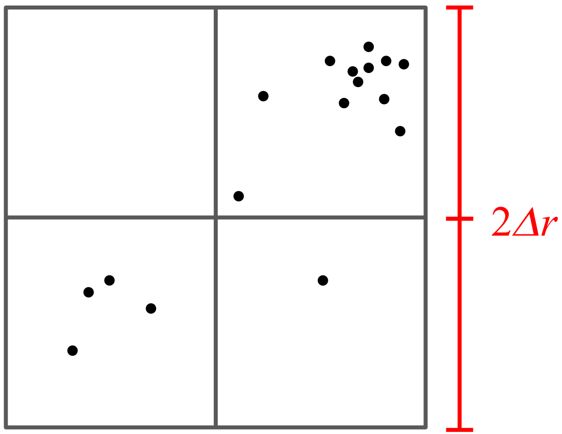
$$m_{\mathrm{cell}} > m_{\mathrm{crit}} = n \times m_{\mathrm{sph}} \times \left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)}\right)^3 \begin{cases} = 1;\ \text{quasi-Lagrangian} & \text{if } \Delta x \geq 2\,\Delta r \\ > 1;\ \text{sub-Lagrangian} & \text{if } \Delta x < 2\,\Delta r \end{cases}$$

Ref

Physical motivation:
We treat the cell as if it's embedded in a volume with a side length of $2\Delta r$



sub-Lagrangian:
Suppress over-refinement at small scales



If $\Delta x_l = 2\Delta r$ then
$m_{\mathrm{crit},l} = 8 \times m_{\mathrm{sph}}$

For next level,
$\Delta x_{l+1} < 2\Delta r$
$m_{\mathrm{crit},l+1} = 64 \times m_{\mathrm{sph}}$

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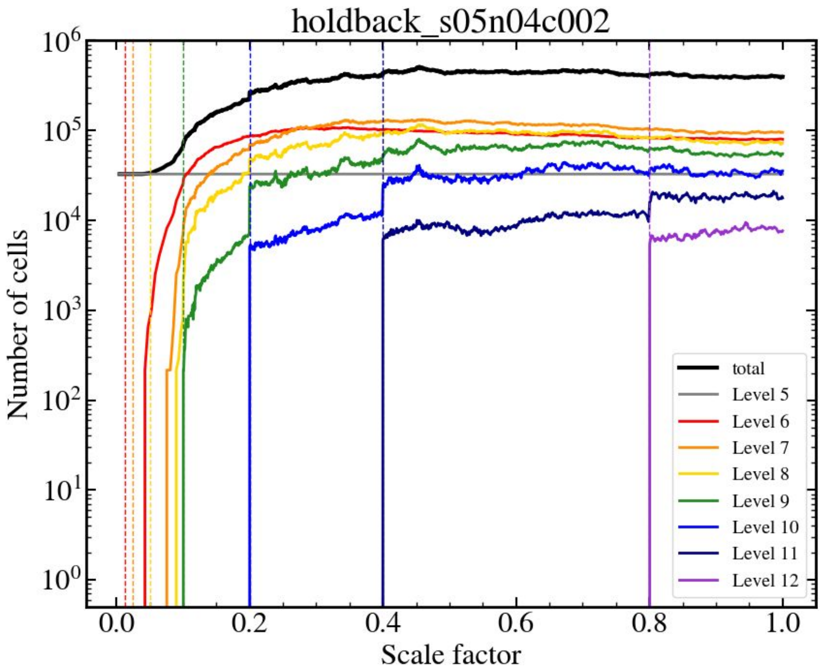
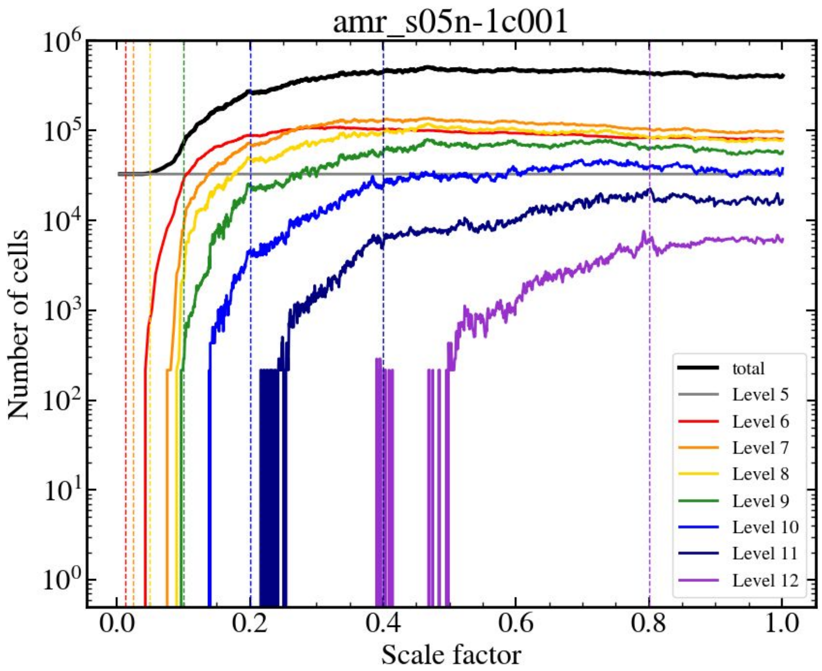
Our solution – 1. Modified Refinement Strategy

PRELIMINARY

Results:

- Public RAMSES
- Small box test: ~ 4 cMpc
- Initial particle (or cell) number: 32^3
- Gravity + Hydro + SF
- No SN or AGN FB





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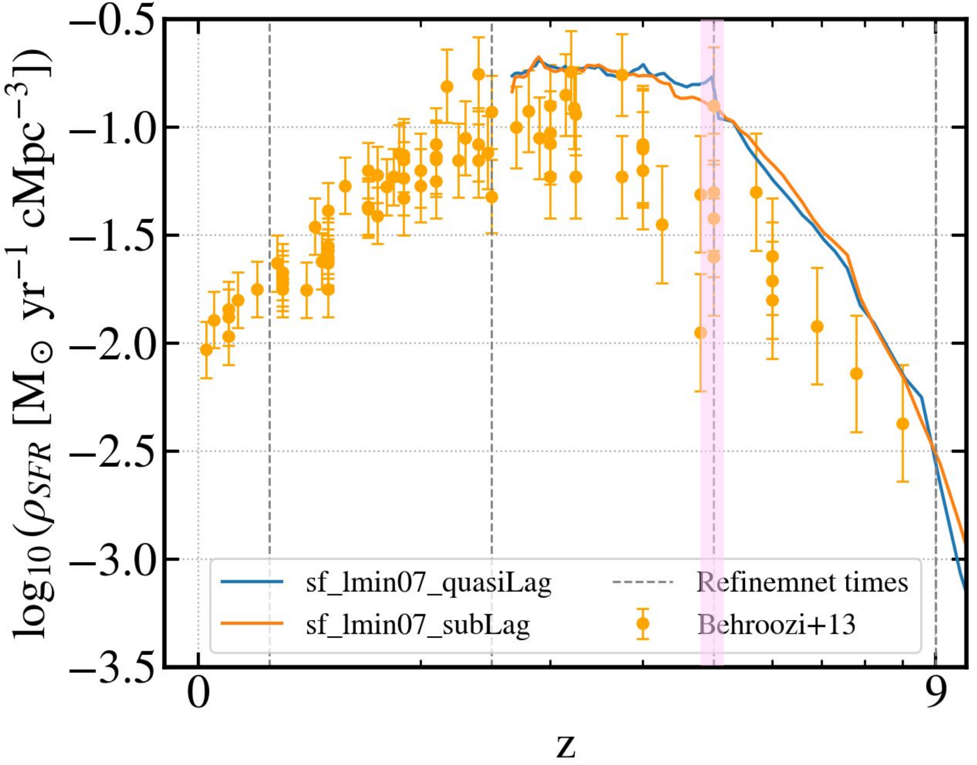
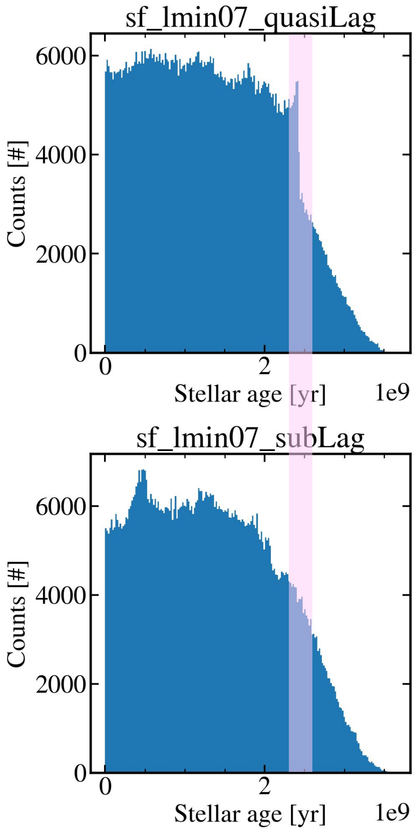
Our solution – 1. Modified Refinement Strategy

PRELIMINARY

Results:





w/ holdback

w/o holdback
our new method

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Our solution – 2. Smoothed Density Field in Poisson Equation

ρ : real density field (computed in PM method)

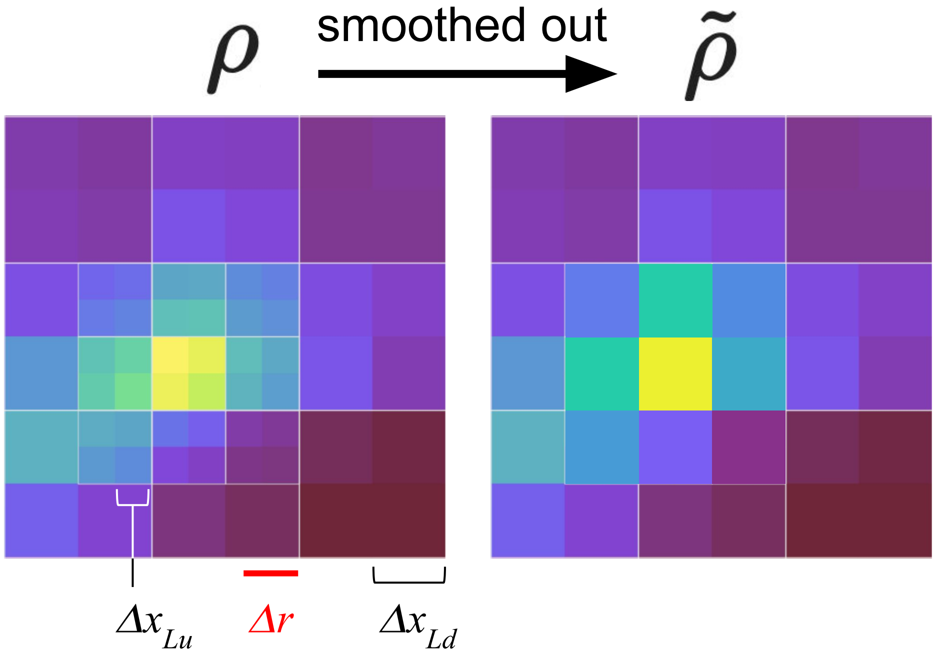
$\tilde{\rho}$: smoothed density field for Poisson equation

Poisson equation: $\nabla^2 \phi = 4\pi G \tilde{\rho}$

$$\tilde{\rho} = \begin{cases} \rho_L, & L \leq L_d, \\ (1-w)\,\rho_{L_u} + w\,\rho_{L_d}, & L \geq L_u \end{cases}$$

$$\Delta x_{L_u} \leq \Delta r < \Delta x_{L_d}$$

where $w = \log_2\left(\frac{\Delta r}{\Delta x_{L_u}}\right) \quad 0 \leq w < 1$



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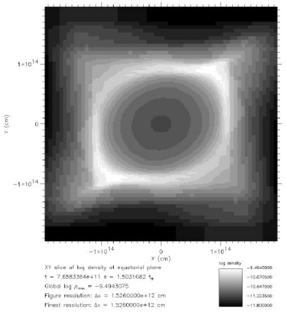
Our solution – 3. Temperature Floor

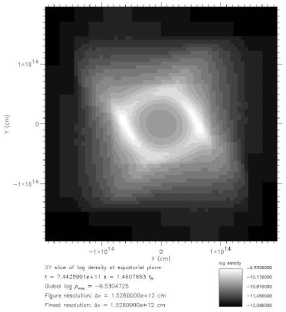
Truelove et al. 1997: $\lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho}} \geq N_J \Delta r$ where $N_J = 4$

artificial heating

N_J=2

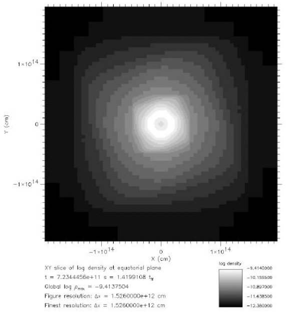


N_J=2.7

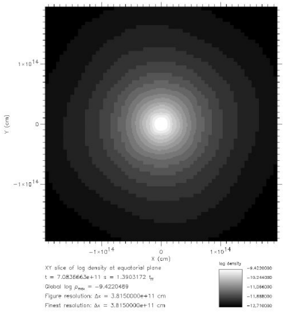


artificial fragment due to the numerical noise

N_J=4

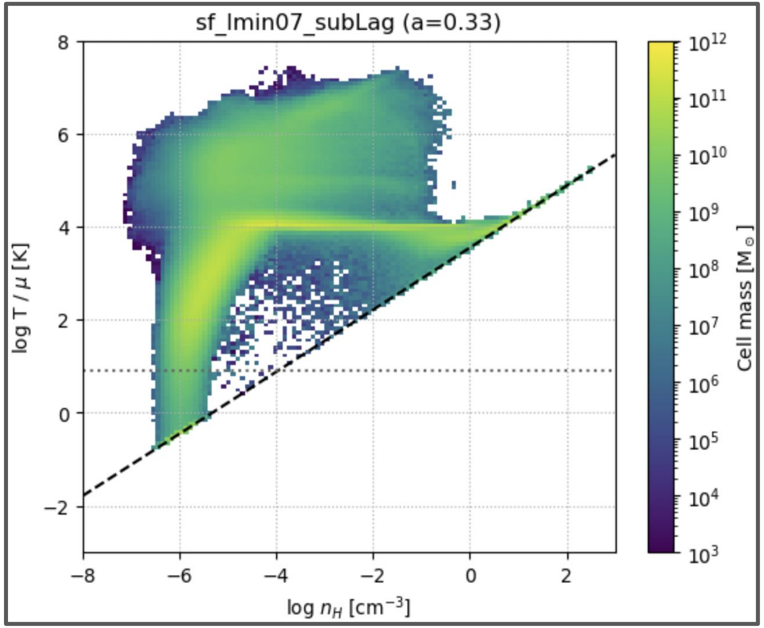


N_J=8



Jeans length should be resolved more than 4 cells

Temperature



Density

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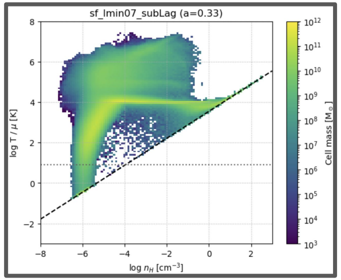
[Summary] Resolving AMR–driven Discontinuities
Implemented in the RAMSES Code

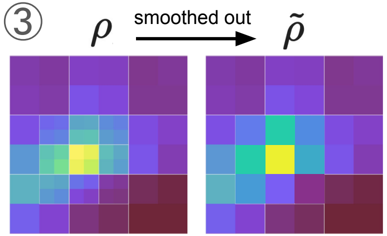
- **Code Development Plan**

① $m_{\rm cell} > m_{\rm crit} = n \times m_{\rm sph} \times \left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)}\right)^3$

① **Modify Refinement Method** [done]
Use a standard quasi–Lagrangian refinement for large cells, but enforces a stricter, sub–Lagrangian criterion once a cell size (Δx) falls below a target resolution (Δr).

②


② **Modify Hydro Solvers** [done]
Use a temperature floor based on Truelove+1997 to prevent artificial fragmentation

③


③ **Modify Gravity Solvers** [almost done]
Use a smooth density field in Poisson equation on scales comparable to the target resolution (Δr)

- **Examine Impact on Galaxy Properties** [future]
Analyze effects on SFR, gas properties, BH accretion, morphology, etc.

STAY TUNED :)

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BACKUP SLIDES

Background Image: a snapshot from Horizon Run 5

Abstract for EANAM10

In cosmological simulations with the AMR code RAMSES, we observe unphysical “jumps” in global quantities like the cosmic star formation rate density (cSFRD). We found these jumps are caused by the “holdback” refinement method, which is used to maintain a constant physical resolution. This method can trigger massive, simultaneous refinement events, which artificially boosts local gas densities and triggers spurious star formation.

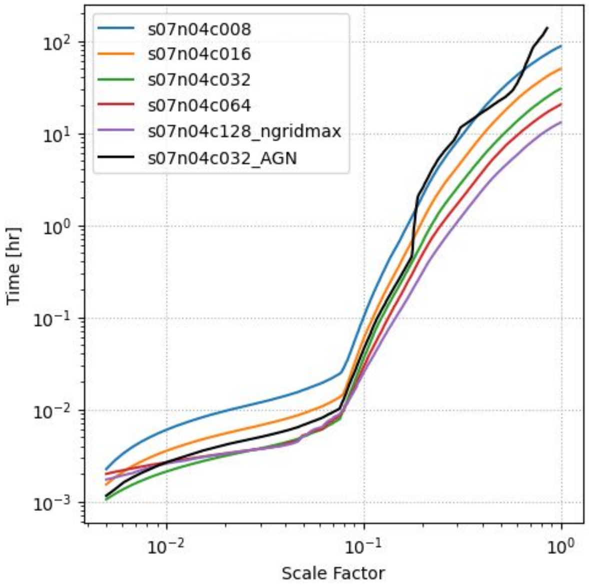
To fix this problem, we are testing a solution with three main parts. First, we modified the refinement strategy itself. Our new hybrid method uses a standard quasi-Lagrangian refinement for large cells, but applies a stricter, sub-Lagrangian rule once a cell’s size (Δx) drops below our target resolution (Δr). This makes it harder for already small cells to refine further and prevents the large, simultaneous refinement events.

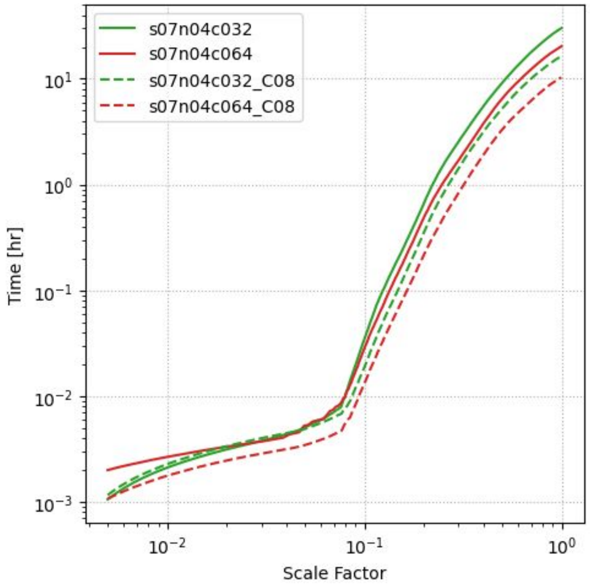
Second, to be consistent with our new refinement scheme, we apply a pressure floor based on the Truelove criterion. This prevents artificial fragmentation by making sure the Jeans length is always properly resolved. Finally, we are working on smoothing the density source for the Poisson solver. This smoothing will also be tied to our target resolution to keep the gravity calculation stable.

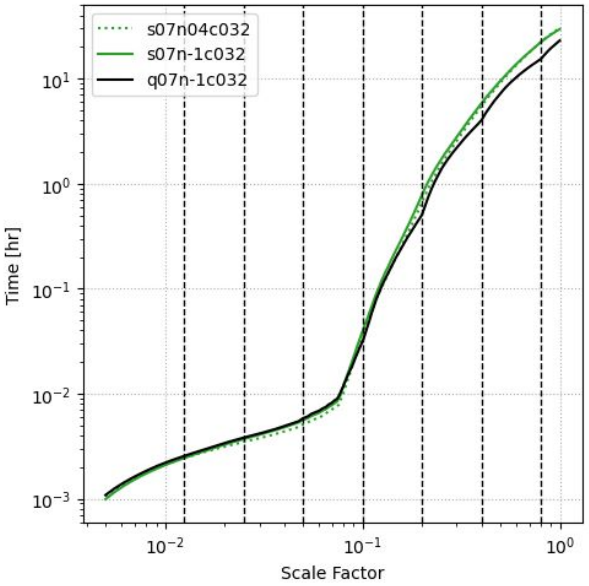
Our initial tests with the first two modifications already show that the artificial jumps in the cSFRD are significantly reduced. At the meeting, we plan to share our latest results from this approach. We hope to receive technical feedback, especially on our ongoing work with the gravity solver.

Performance Test: Running Time

PRELIMINARY







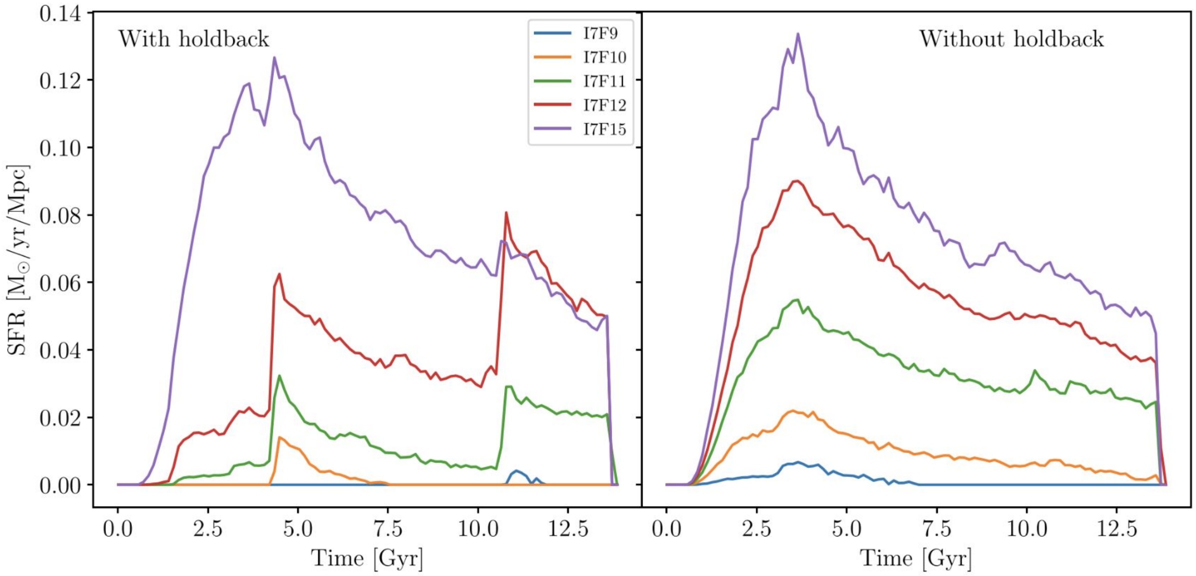
s07n04c032	108,511 sec
s07n04c032_C08	59,079 sec
s07n04c064	73,196 sec
s07n04c064_C08	37,096 sec

q07n-1c032	83,104 sec
s07n-1c032	106,610 sec
s07n04c032	108,511 sec

How Can We Reduce It?

Snaith et al. 2018 (RAMSES)

- **Turn off holdback** ⇒ leads to **unmatched resolution** with subgrid models (tuned for ~ 1 pkpc)



fixed physical resolution **fixed comoving resolution**

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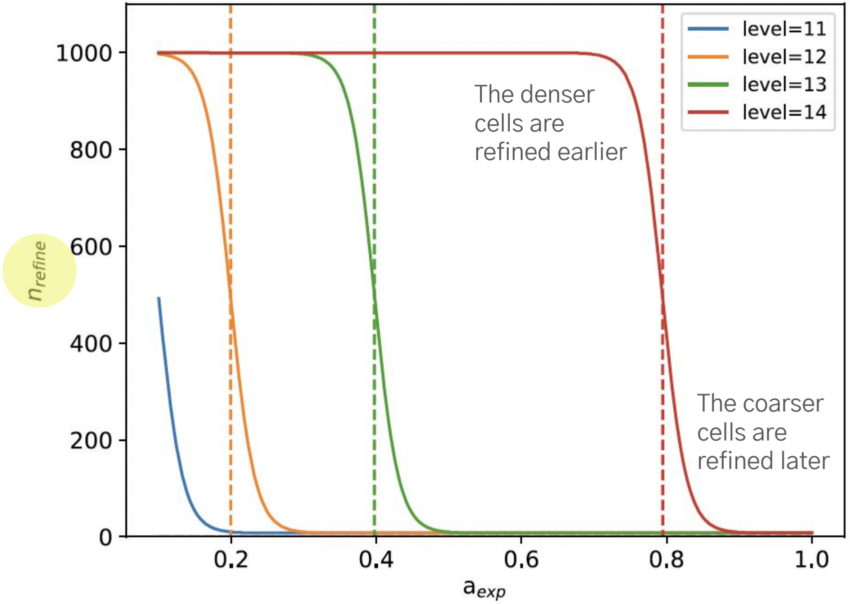
How Can We Reduce It?

$$m_{\rm cell} > m_{\rm crit} = n \times m_{\rm sph}$$

Snaith et al. 2018 (RAMSES)

- Turn off holdback ⇒ leads to **unmatched resolution** with subgrid models (tuned for ~ 1 pkpc)
- Keep holdback, but smooth the refinement ⇒ **ad hoc, lacks physical motivation**



transition more gradually. The prescription of grid release used in our modified version of RAMSES uses a Logistic function of the form

$$n_{\rm refine}(a) = n_{\rm final} + (n_{\rm max} - n_{\rm final})\left(1 - \frac{1}{1+\exp^{-S(a-c)}}\right) \quad (6)$$

where S sets the steepness of the slope, c is the epoch at which the refinement level is released, $n_{\rm refine}$ is the current number of particles in a cell required for refinement under the quasi-Lagrangian approach, and $n_{\rm final}$ is the final number of particles required for refinement, usually eight (Dubois et al. 2014).

We take c from the current RAMSES code,

$$c = 4^{1/n_{\rm d}} 2^{-(l_{\rm max} - l_i)}, \quad (7)$$

where $n_{\rm d}$ is the number of dimensions, $l_{\rm max}$ is the maximum refinement level, and l_i is the i-th refinement level. Using these equations, the grid release occurs more gradually, with a rate that depends on the value of S.

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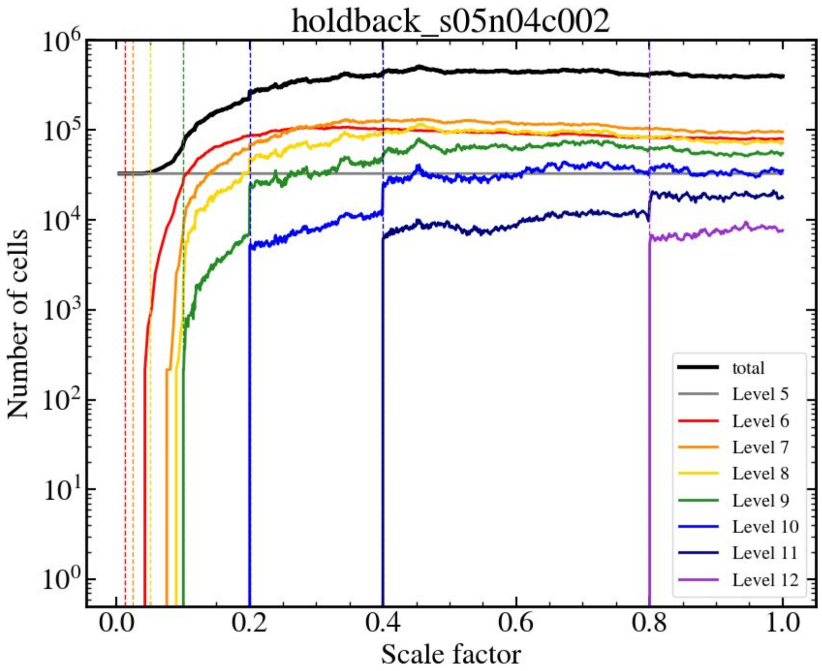
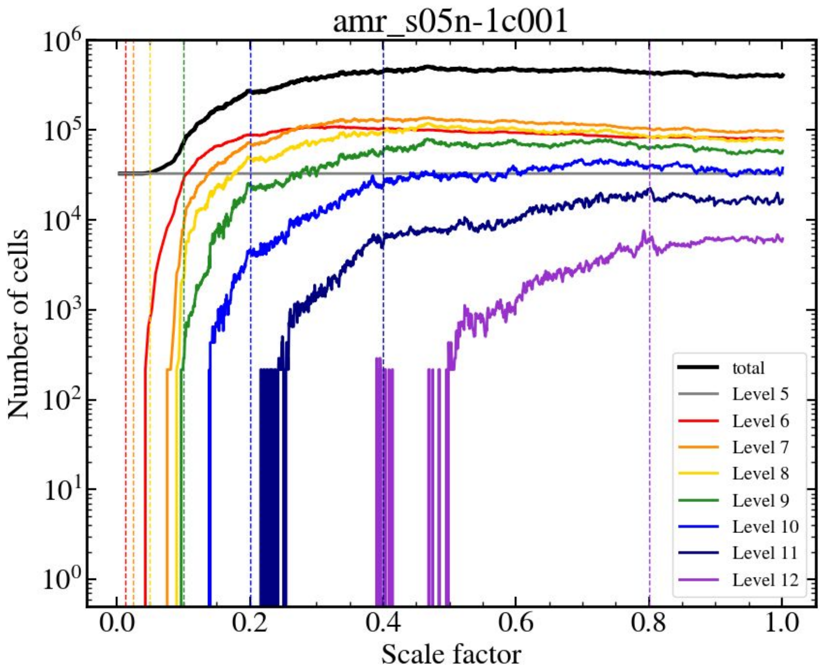
Our solution – 1. Modified Refinement Strategy

PRELIMINARY

Cell count over time

- Public RAMSES
- Small box test: ~ 4 cMpc
- Initial particle (or cell) number: 32^3
- Gravity + Hydro + SF
- No SN or AGN FB





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Our solution – 1. Modified Refinement Strategy

PRELIMINARY

Cell count over time

- Public RAMSES
- Small box test: ~ 4 cMpc
- Initial particle (or cell) number: 32^3
- Gravity + Hydro + SF
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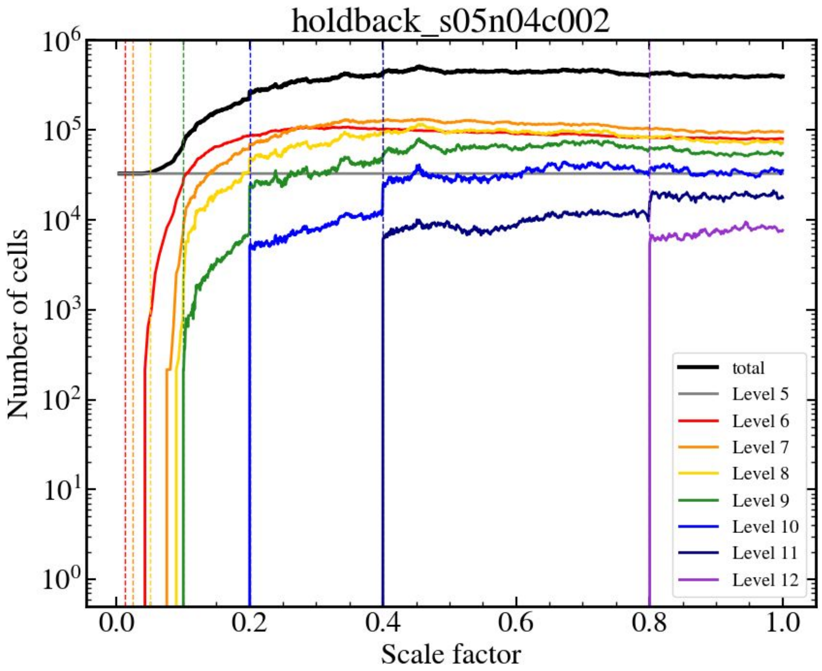

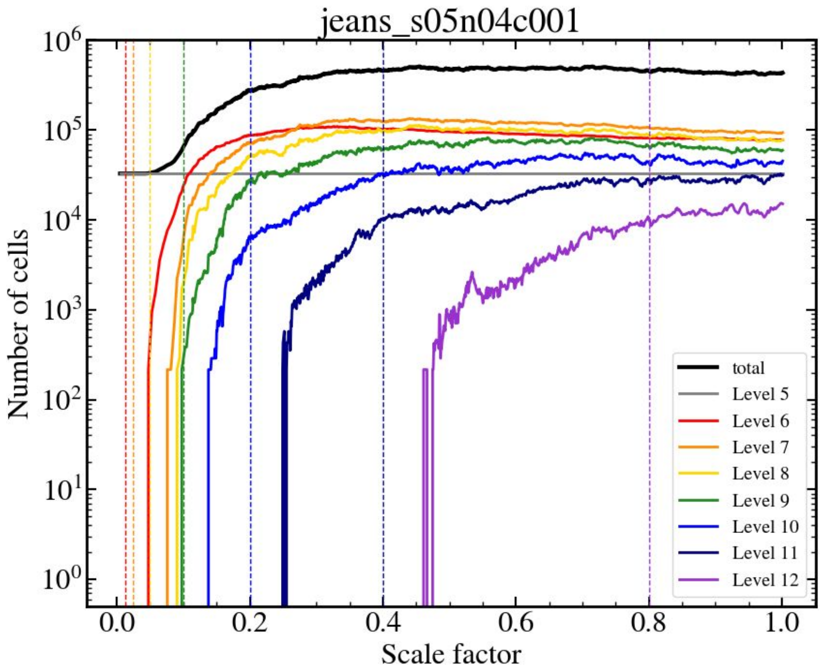

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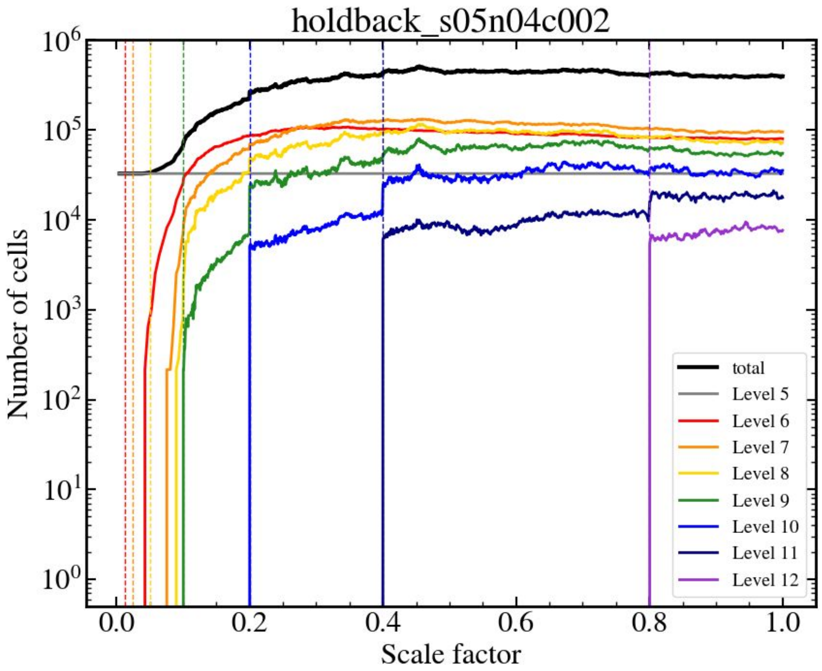

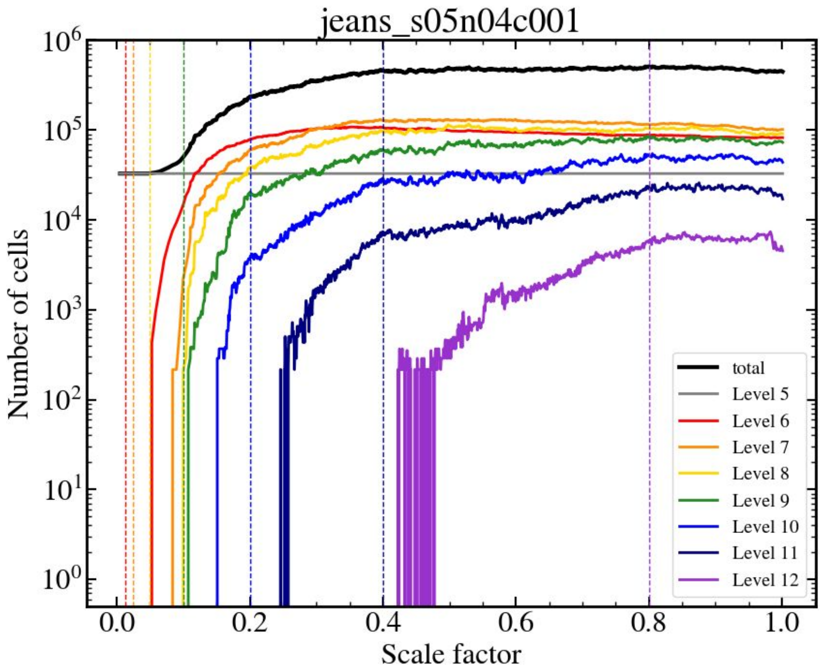

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Mass–based Refinement Approaches

$$m_{\text{cell}} > m_{\text{crit}}$$

Method	Condition	Refinement Tendency	In RAMSES?
Quasi-Lagrangian	$m_{\text{crit}} = n \times m_{\text{sph}}$	Standard	✅ Yes
Super-Lagrangian	$m_{\text{crit}} < n \times m_{\text{sph}}$	Easier to refine	❌ No
Sub-Lagrangian	$m_{\text{crit}} > n \times m_{\text{sph}}$	Harder to refine	❌ No

How much should it be increased
in a physically meaningful way?

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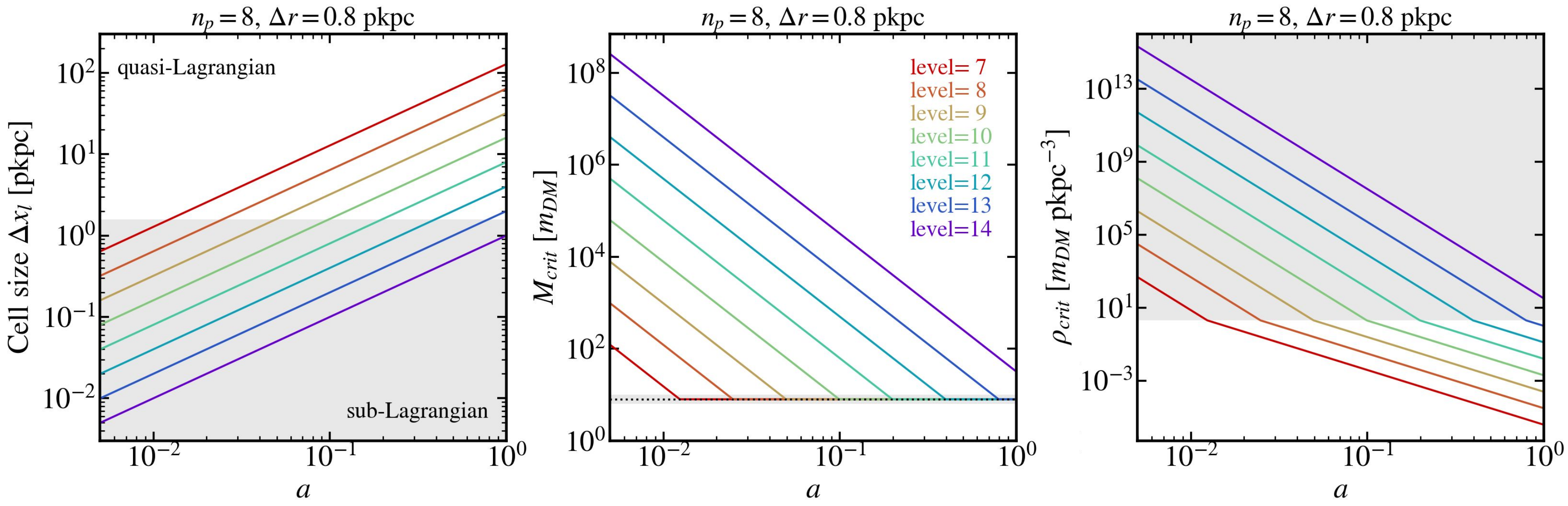
Our Approach

No holdback, but modified refinement

We treat the cell as if it's embedded in a volume associated with $2\Delta r$, so refinement becomes harder when the actual cell size Δx is already smaller than $2\Delta r$.

if $n = 8$

$$m_{\rm cell} > m_{\rm crit} = n \times m_{\rm sph} \times \left(\frac{2\Delta r}{\min(\Delta x, 2\Delta r)} \right)^3 \begin{cases} = 1;\ \text{quasi-Lagrangian} & \text{if } \Delta x \geq 2\,\Delta r \\ > 1;\ \text{sub-Lagrangian} & \text{if } \Delta x < 2\,\Delta r \end{cases}$$



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Refinement Methods in *RAMSES* [link]

In RAMSES, the refinement strategy is governed by several criteria, which can be applied individually or in combination. Key refinement criteria include:

1. **Mass-based Refinement (Quasi-Lagrangian Method)**:
   Cells are refined when the mass of baryonic matter exceeds a certain threshold, or when the number of dark matter particles in a cell surpasses a specified count. This method maintains **consistent mass resolution** throughout the simulation.

2. **Discontinuity-based (=Gradients-based) Refinement**:
   This method refines cells based on the local gradients of certain physical quantities (e.g., pressure, density, Mach number, …), ensuring better resolution in regions with sharp changes, like shock fronts or contact discontinuities. A similar criterion based on second derivatives (Laplacian) has also been implemented.

3. **Geometry-based Refinement**:
   This method refines specific regions based on their geometric properties. Users can define the center, size, and shape parameters of the region to be refined at each AMR level.

4. **Jeans Length Refinement**:
   To accurately capture gravitational instabilities, cells are refined if their size exceeds the local Jeans length divided by a specified factor. This ensures that critical physical scales are adequately resolved.

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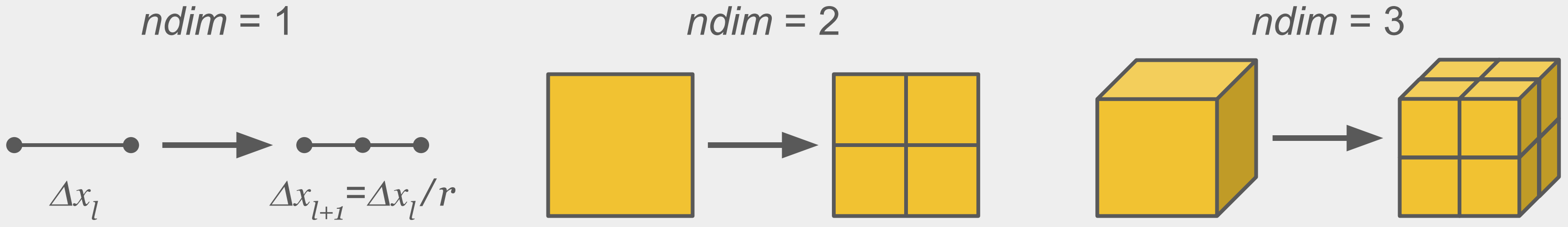

- refinement factor: $r = \Delta x_l / \Delta x_{l+1} = 2$
- # of child cells = r^{ndim}
- Mass threshold: $M_{\rm cell} > M_{\rm crit} = n\, m_{\rm DM}$ (where $n = r^{ndim}$) ; constant
- Density threshold: $\rho_{\rm cell} > \rho_{\rm crit} = M_{\rm crit} / \Delta x_l^{ndim}$; level-dependent