A jet-like configuration with force-free combined magnetic field around a black hole

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Force-free solution

The plasma density is higher than Goldreich-Julian density.
$(\rho > \rho_{GJ})$

The external electric field is screened.

The energy-momentum density of the electromagnetic field is much higher than the energy-momentum density of the plasma.
$\left(T^{\mu\nu}{}_{EM} \gg T^{\mu\nu}{}_{plasma}\right)$

$$\nabla_\mu T^{\mu\nu}{}_{EM} = 0$$

From Maxwell equation and the axisymmetric stationary condition, one can get the GR Grad-Shafranov equation.
(Blandford-Znajek 1977)

GR Grad-Shafranov equation(GSE)

One can get force-free solution for
the axisymmetric stationary condition.

$$\frac{\Sigma B_T {B_T}'}{\Delta \sin\theta} = \frac{\omega^2}{c^2}\alpha + 2\frac{\omega}{c}\beta + \gamma + \frac{\sin\theta}{\Sigma\Delta}\left(A\frac{\omega}{c} - 2rr_g a\right)\left(\Delta\left(\partial_r A_\phi\right)^2 + \left(\partial_\theta A_\phi\right)^2\right)\frac{\omega'}{c}$$

$$\alpha = \sin\theta\,\partial_r\left(\frac{A}{\Sigma}\partial_r A_\phi\right) + \Delta^{-1}\partial_\theta\left(\frac{A\sin\theta}{\Sigma}\partial_\theta A_\phi\right)$$

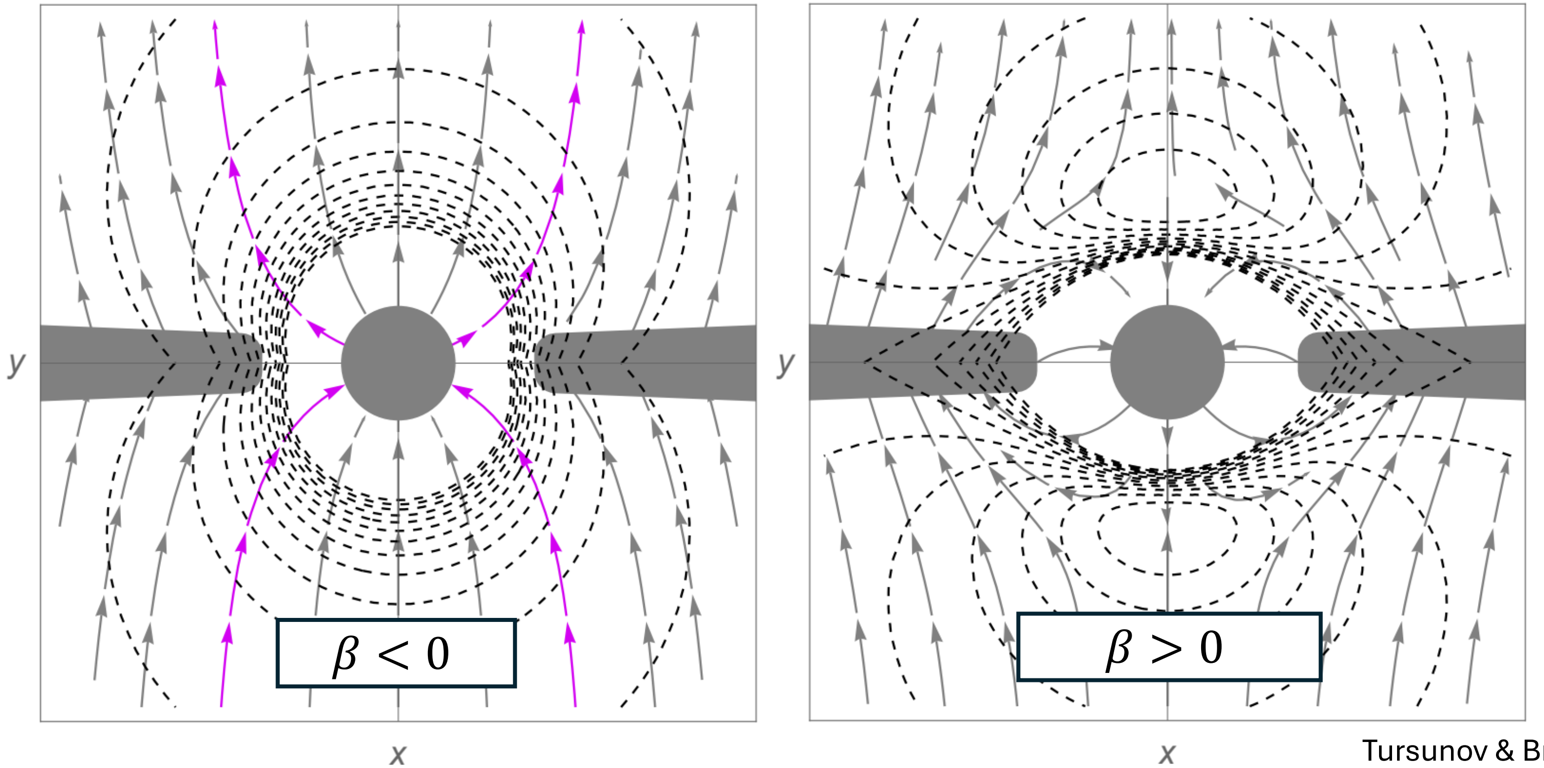
$$\beta = -2r_g a\left[\sin\theta\,\partial_r\left(\frac{r\partial_r A_\phi}{\Sigma}\right) + \frac{r}{\Delta}\partial_\theta\left(\frac{\sin\theta\,\partial_\theta A_\phi}{\Sigma}\right)\right]$$

$$\gamma = -\partial_r\left(\frac{\Sigma - 2r_g r}{\Sigma\sin\theta}\partial_r A_\phi\right) - \partial_\theta\left(\frac{\Sigma - 2r_g r}{\Sigma\Delta\sin\theta}\partial_\theta A_\phi\right)$$

$\omega(A_\phi)$: electromagnetic angular velocity

$B_T(A_\phi) = \frac{\Delta}{\Sigma}\sin\theta\, B_\phi$

Combined magnetic field



Tursunov & Britzen 2024

$$A_\phi = \frac{1}{2}B_0 g_{\phi\phi} + \left(aB_0 - \frac{Q}{2M}\right)g_{\phi t} + \frac{(r^2 + a^2)}{r^2 + a^2\cos^2\theta}P|\cos\theta|$$

$$P = \frac{B_0}{2}\boldsymbol{\beta}{r_g}^2, \quad r_g = \frac{GM}{c^2}$$

Light surface (singularity)

$$1 - \frac{\omega^2}{c^2}\frac{A\sin^2\theta}{\Sigma} + 4\frac{\omega}{c}\frac{r_g r a \sin^2\theta}{\Sigma} - \frac{2r_g r}{\Sigma} = 0$$

The positions (r, θ) where are satisfied with the above condition are light surfaces(LSs).

Also, left part of the above equation is the coefficient of second derivatives of GR GSE. → The LS is the singularity of GR GSE.

Proper ω and B_T smooth A_ϕ at LSs.

Depending on initial and boundary conditions, there are two or one LS.

To make just one light surface, we set the range of parameters

$$a_* \lesssim 0.95,\ \beta \gtrsim -50$$

Initial condition

We use the combined magnetic field to the initial condition of our calculations.

$\beta < 0$: jet-like configulation

We set that the jet-wall boundary is the magnetic field line which is connected to the horizon at $\theta = \pi/2$.

We transform $r \rightarrow R = (r - r_{BH}) / (r + r_H)$,
$\theta \rightarrow \Theta = \theta / \theta_{wall}(r)$.

Initial condition

Normalization of A_ϕ

$$a_{\phi,n} = (A_\phi - P)/(A_{\phi,\mathrm{wall}} - P) = a_\phi/a_{\phi,\mathrm{wall}}$$

Normalization of ω and B_T

$$\omega_n = \omega/\Omega_{BH}\,, \qquad \Omega_{BH} = ac\,/\,({r_{BH}}^2 + a^2)$$

$$B_{T,n} = B_T c\,/\,\Omega_{BH} a_{\phi,\mathrm{wall}}$$

The initial conditions of ω and B_T from Znajek condition

$$B_T[A_\phi(r_{BH},\theta)] = \frac{\sin\theta\left[\frac{\omega}{c}({r_{BH}}^2 + a^2) - a\right]}{{r_{BH}}^2 + a^2\cos^2\theta}\,\partial_\theta A_\phi(r_{BH},\theta)$$

Boundary condition

BH rotational axis boundary

$$a_{\phi,n} = 0, \quad \omega_n = 0.5, \quad B_{T,n} = 0$$

Jet-wall boundary

$$a_{\phi,n} = 1$$

At infinity, if the jet is sufficiently collimated,

$$A_\phi = \frac{1}{2} B_0 r^2 \sin^2\theta + P \quad \rightarrow \quad a_{\phi,n} = \frac{B_0 r^2 \sin^2\theta}{2 a_{\phi,\mathrm{wall}}}$$

→ Dirichlet boundary

Constrain and regularity condition

Constrain condition

At infinity, GR GSE -> non-relativistic GSE

If the magnetic field configuration at infinity is parallel to z-axis,

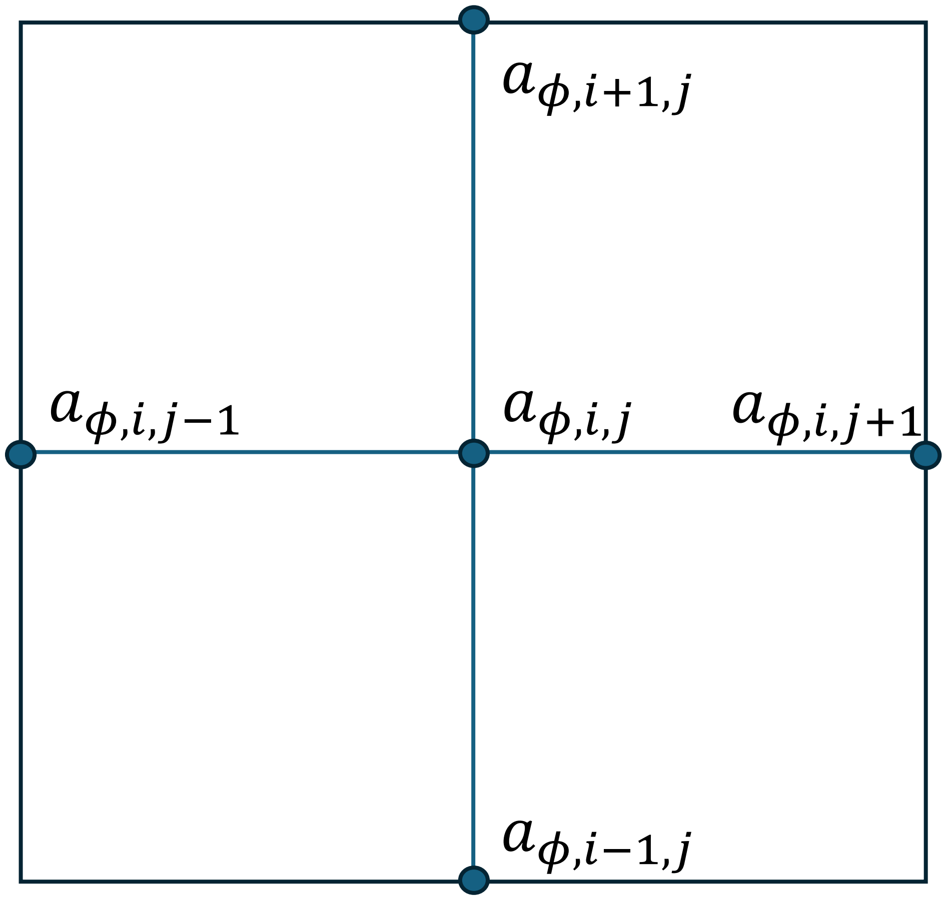
$$B_T(A_\phi) = -2\frac{\omega(A_\phi)}{c}(A_\phi - P)$$

Nathanail et al. 2014
Pan et al. 2017

At the horizon, there is the regularity condition which is the Znajek condition.

$$B_T[A_\phi(r_{BH},\theta)] = \frac{\sin\theta\left[\frac{\omega}{c}({r_{BH}}^2 + a^2) - a\right]}{{r_{BH}}^2 + a^2\cos^2\theta}\partial_\theta A_\phi(r_{BH},\theta)$$

Numerical calculation

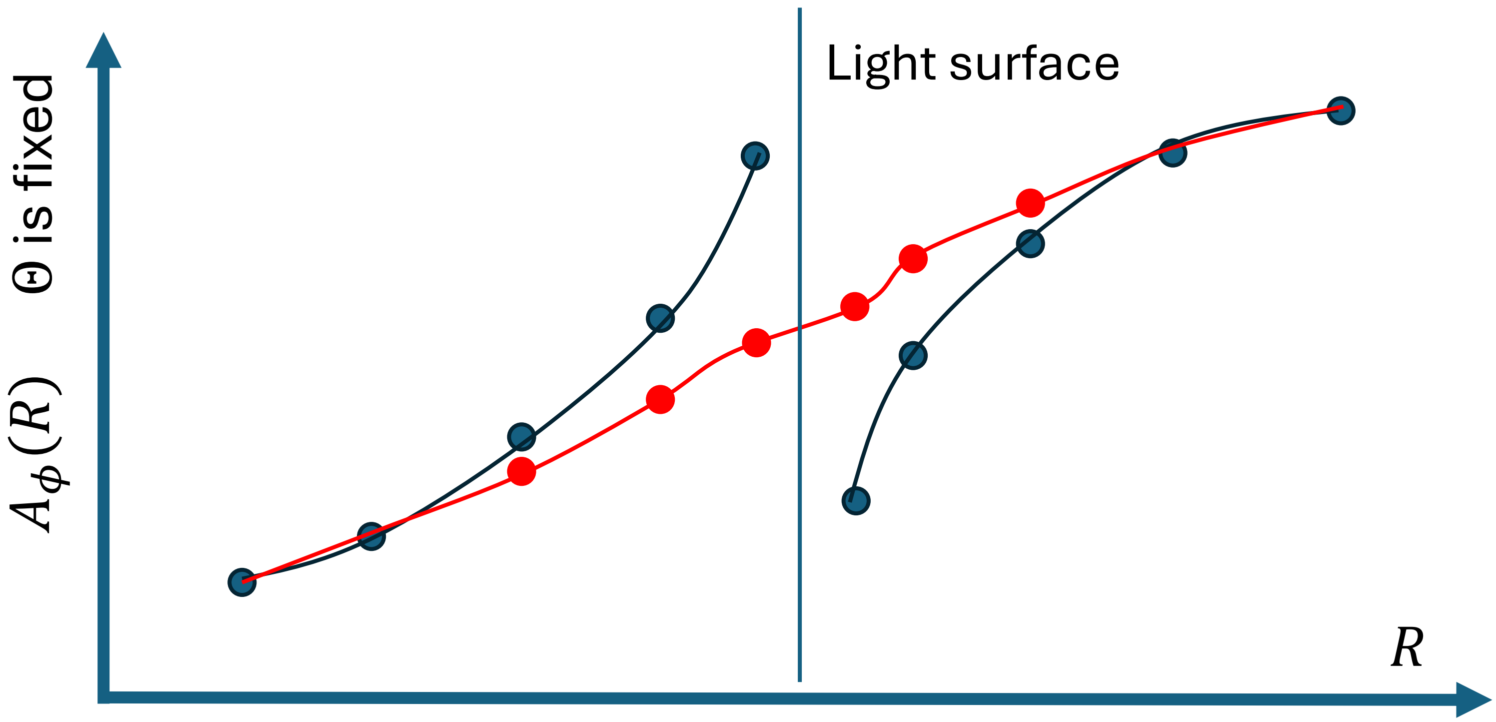


Second order centered
Finite difference discretization

The under-relaxation iteration method (update A_ϕ)

Every iteration, using some technics, update ω.

Numerical calculation

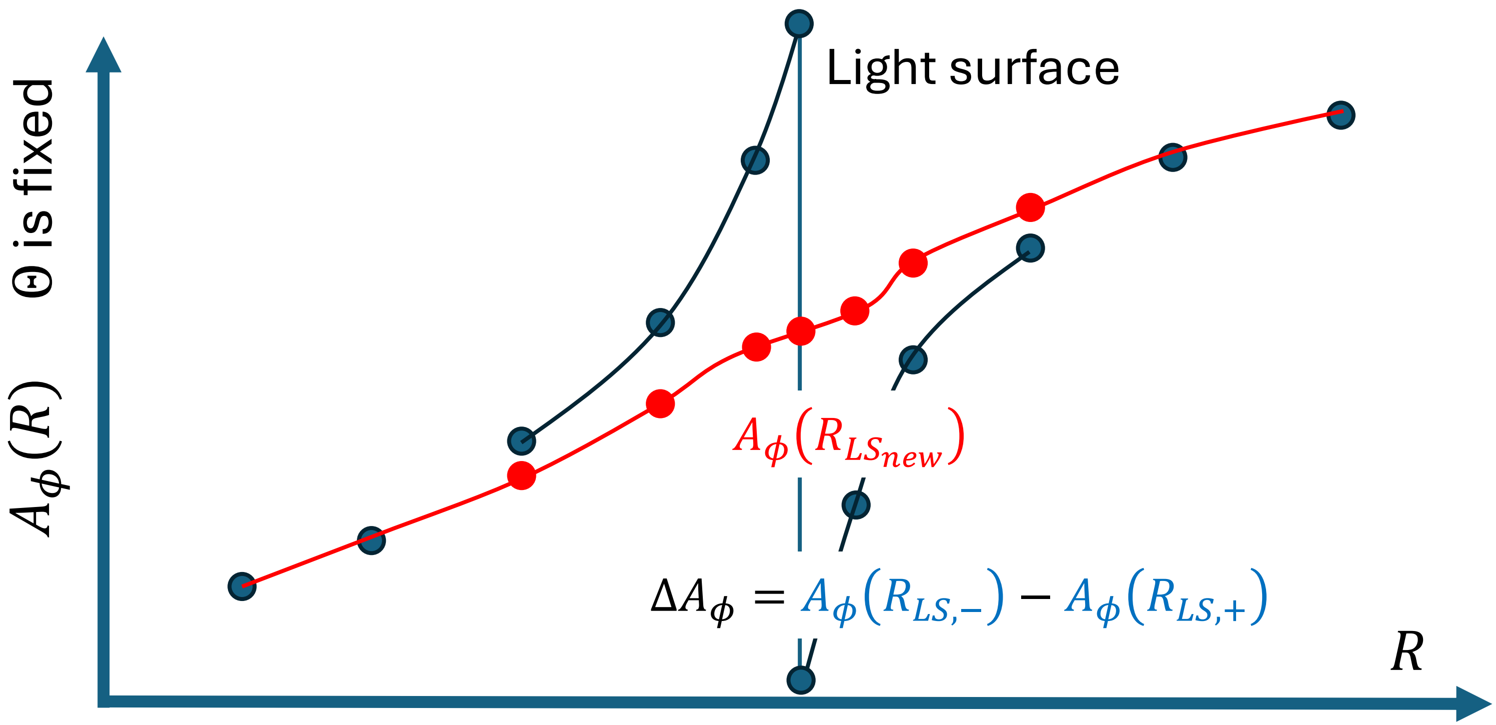


For the same Θ line, along the radius R, we find out where the LS is.

We remove 3 points each before and after the LS.

We interpolate the gap, and regenerate total 6 points.

Numerical calculation

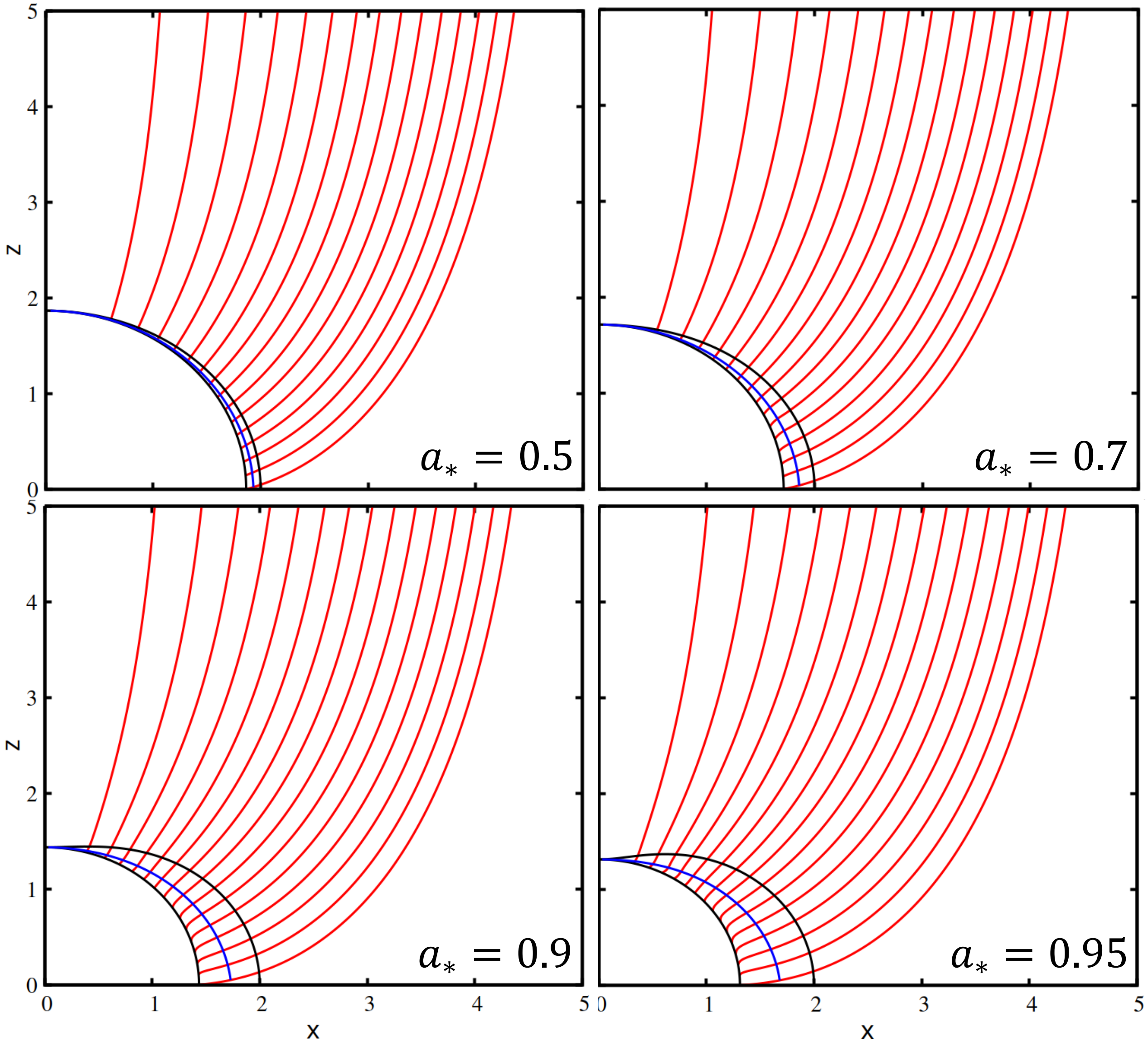


We extrapolate old 3 points each before and after the LS respectively to the LS.

$$\omega_{new}(A_{\phi,LS_{new}}) = \omega_{old}(A_{\phi,LS_{old}}) - f\Delta A_\phi$$

Contopolos et al. 2013
Nathanail et al. 2014
Pan et al. 2017

Result



$\beta = 0, \ Q = 2aMB_0$

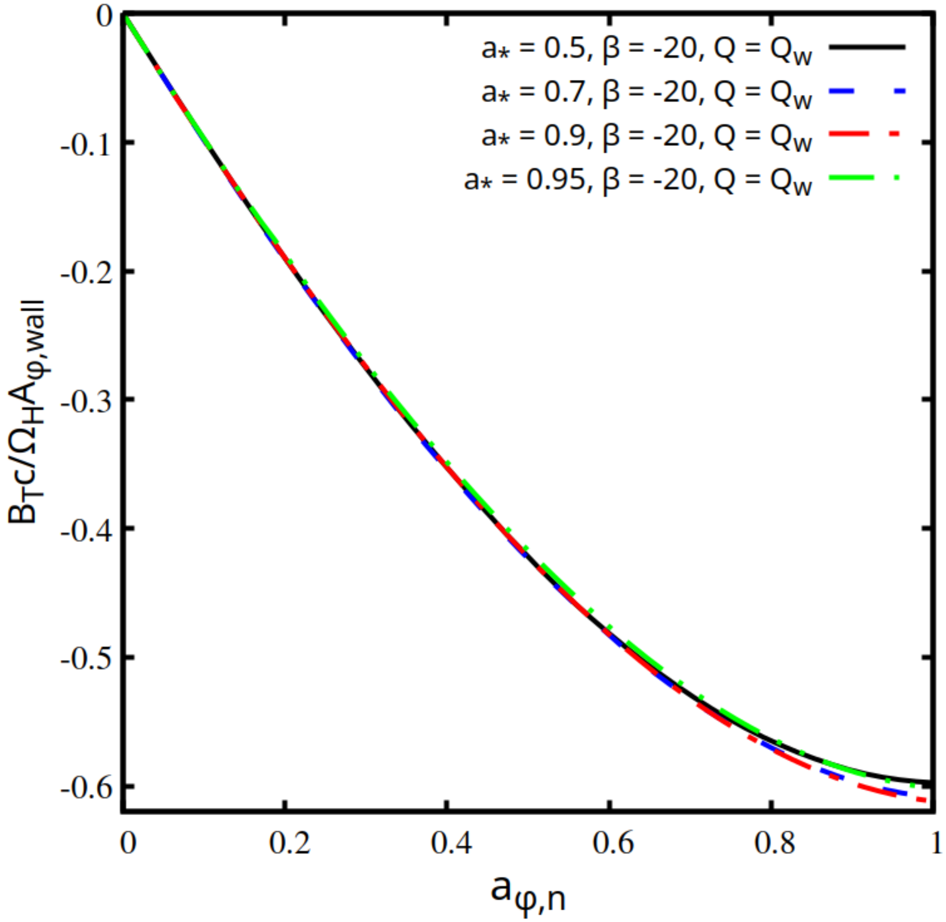
Horizon

Ergosphere

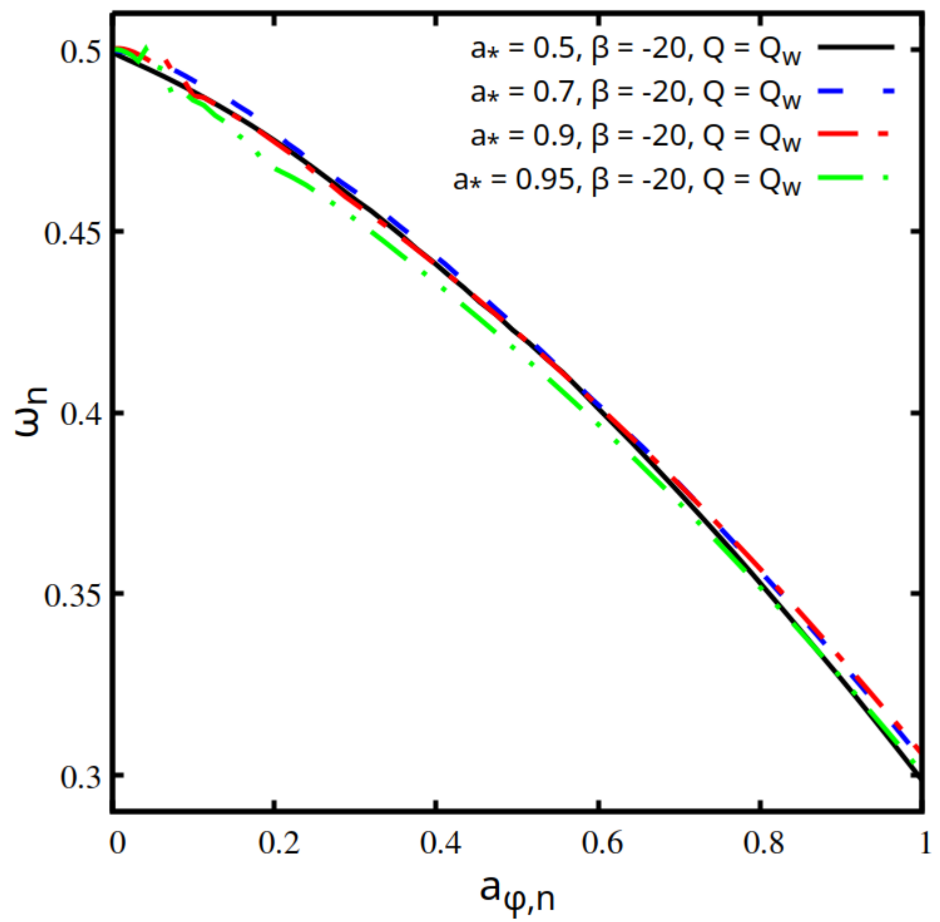
Magnetic field line

Light surface

Result

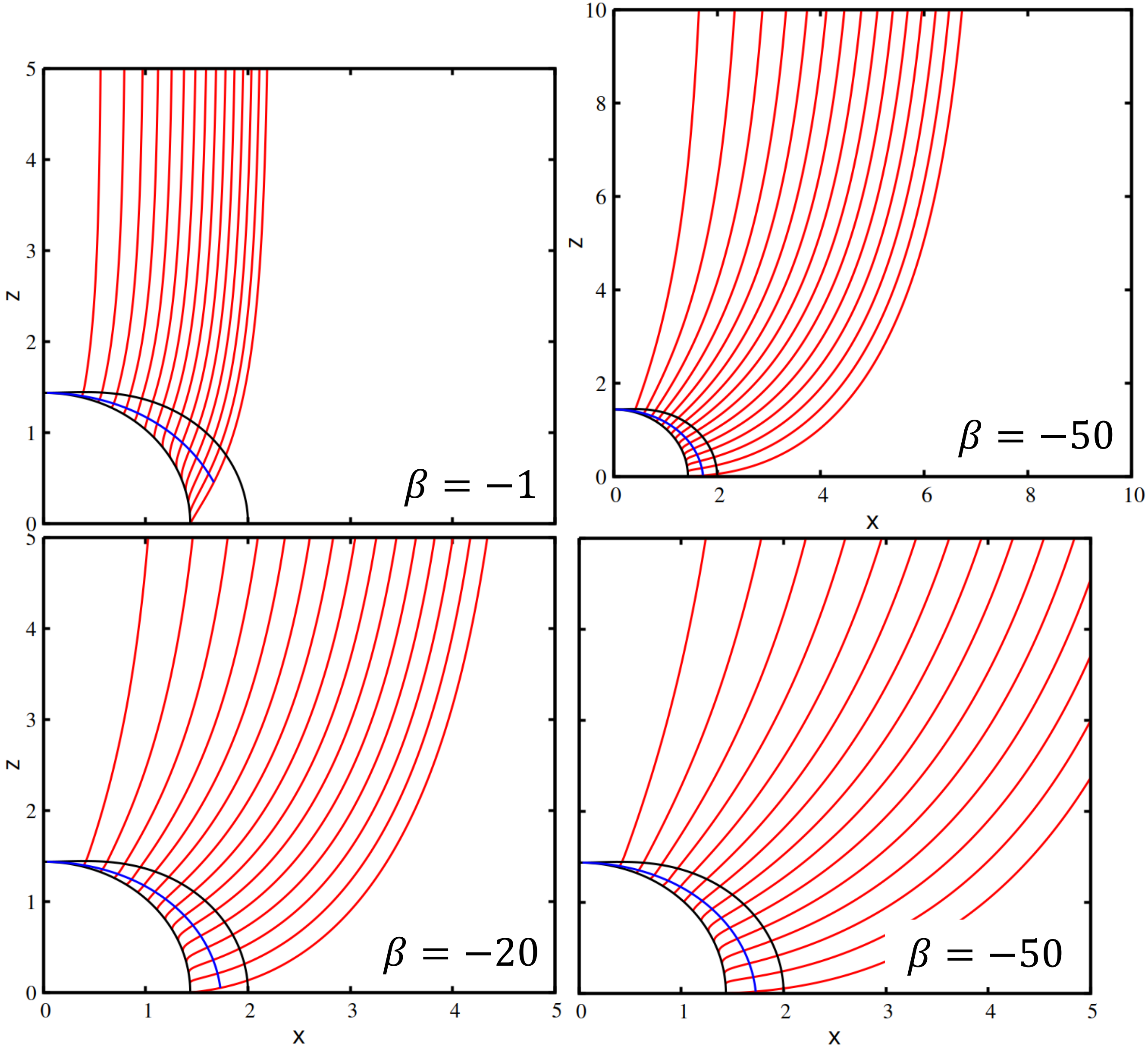


$a_* = 0.5$, $\beta = -20$, $Q = Q_w$
$a_* = 0.7$, $\beta = -20$, $Q = Q_w$
$a_* = 0.9$, $\beta = -20$, $Q = Q_w$
$a_* = 0.95$, $\beta = -20$, $Q = Q_w$
$B_T c/\Omega_H A_{\varphi,wall}$
$a_{\varphi,n}$
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0.2
0.4
0.6
0.8
1



$a_* = 0.5$, $\beta = -20$, $Q = Q_w$
$a_* = 0.7$, $\beta = -20$, $Q = Q_w$
$a_* = 0.9$, $\beta = -20$, $Q = Q_w$
$a_* = 0.95$, $\beta = -20$, $Q = Q_w$
ω_n
$a_{\varphi,n}$
0.5
0.45
0.4
0.35
0.3
0
0.2
0.4
0.6
0.8
1

Result



$a_* = 0.9,$

$Q = 2aMB_0$

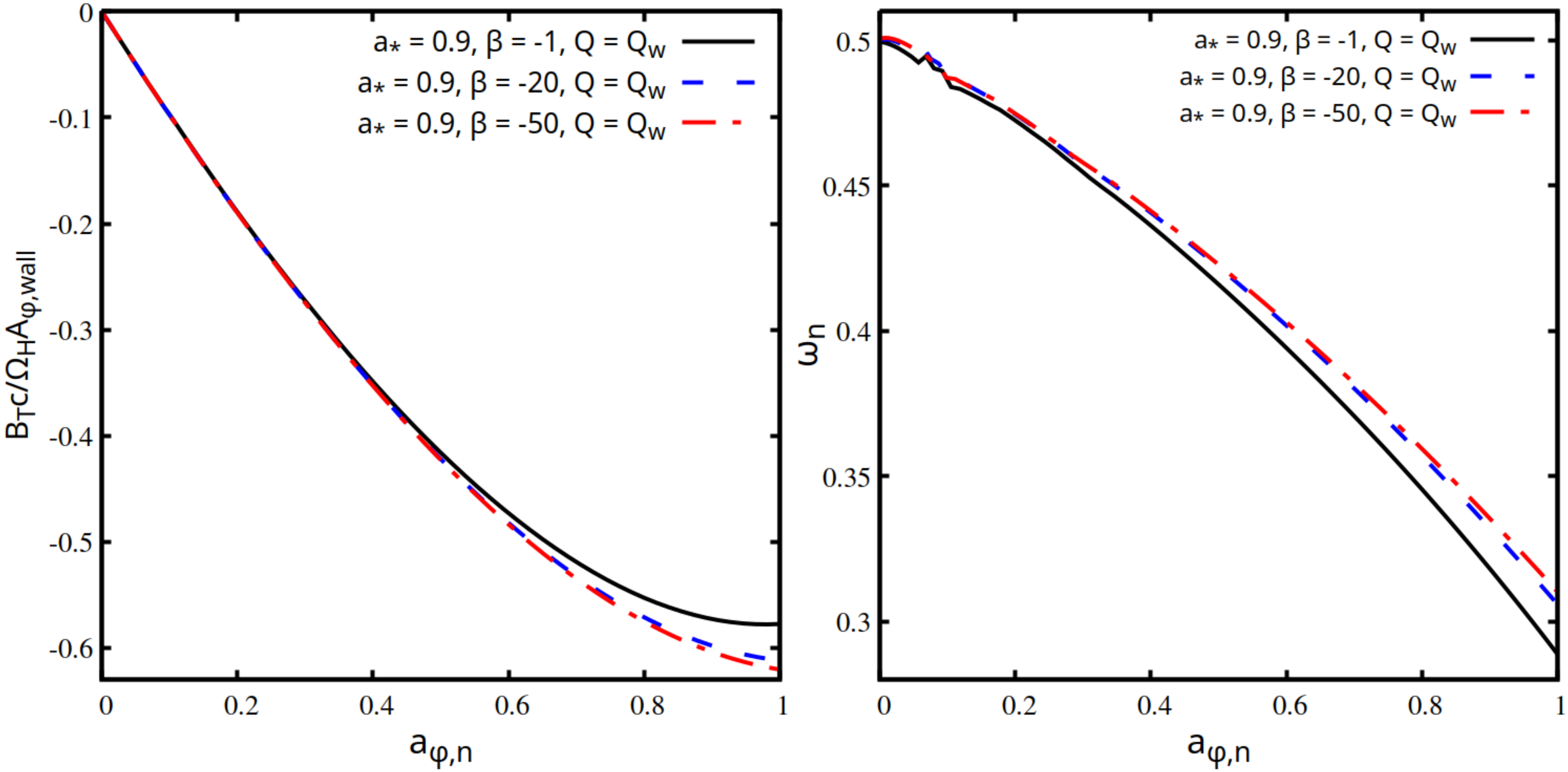
Horizon

Ergosphere

Magnetic field line

Light surface

Result



a∗ = 0.9, β = -1, Q = Qw
a∗ = 0.9, β = -20, Q = Qw
a∗ = 0.9, β = -50, Q = Qw
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
BTc/ΩHAφ,wall
0
0.2
0.4
0.6
0.8
1
aφ,n
a∗ = 0.9, β = -1, Q = Qw
a∗ = 0.9, β = -20, Q = Qw
a∗ = 0.9, β = -50, Q = Qw
0.5
0.45
0.4
0.35
0.3
ωn
0
0.2
0.4
0.6
0.8
1
aφ,n

Summary

We consider the combination of vertical uniform and split monopole field in Kerr spacetime to initial and boundary conditions of GR Grad-Shafranov equation.

We set $a_* \lesssim 0.95$ and $0 > \beta \gtrsim -50$ to get the jet-like configuration and to determine just one light surface.

We obtain $B_T(A_\phi)$ from the constrain condition and use some technics of smooth of A_ϕ.

There are not much difference of $\omega(A_\phi)$ and $B_T(A_\phi)$ by a_*, Q. But $\omega(A_\phi)$ is smaller, if the field configuration resemble the vertical uniform field